MODELING SINGLE SLASH PINE TREE GROWTH
AND SPACING RELATIONSHIPS

BY

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

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This study deals with modeling the effects of competition and
available growing space on the growth and development of individual
slash pine trees over time. A model to predict the cumulative growth of
diameter at breast height, total height, cubic volume, and bole surface
area has been developed using a modified form of the Verhulst growth
function.

In the past, single-equation estimation procedures, such as
Ordinary Least Squares (OLS), have been employed to estimate the
structural coefficients associated with a system of equations. This
study advocates Three-Stage Least Squares (3SLS), which allows
estimation of the structural coefficients simultaneously using a systems
approach. OLS and 3SLS are examined under two assumptions concerning
the correlation of residual errors: contemporaneous covariance and
serial correlation.

The validity of the models was scrutinized following data-splitting (cross-validation) and comparison of the models against data fit and prediction accuracy. In addition, the signs of the estimated coefficients were compared with theoretical expectations. Parameter estimates for each procedure were obtained from data accumulated in a spacing study which was established in 1962 and remeasured approximately every two years for the past 21 years.

Results of this study indicate that the suggested growth model depicts the actual process quite well by using either OLS or 3SLS. The structural equations had a weighted $k**2$ value of approximately 93 percent. However, the 3SLS procedure was more efficient in estimating the coefficients of the growth model than OLS. The OLS procedure does not take into consideration the correlation of errors among equations or the presence of endogenous variables as explanatory ones. This results in a specification error which affects the variance-covariance matrix used in the estimation procedure.

Both OLS and 3SLS yield unbiased estimates of the model parameters. However, only the latter procedure assures asymptotically efficient parameter estimates with respect to the identification of the variance-covariance matrix. These two desirable properties of 3SLS lend validity to the growth model, allow for hypothesis testing, and construction of confidence intervals.

INTRODUCTION

Recently, a renewed interest has emerged in modeling the effects of growing space and competition among individual trees within a forest plantation. Two basic approaches have been used: stand descriptive models and single tree models. In stand models, the variation associated with tree competition is evaluated indirectly in terms of stand averages and stocking densities. While such characteristics are useful under certain conditions, they tend to obscure the cause and effect relationship among neighboring trees.

If one considers single trees as the major component of the system, stand development may be modeled as the interaction between growth and competition of individual trees (Daniels et al. 1979). This feature allows the use of both tree and stand parameters to describe the interactions among competing neighborhood trees.

Models incorporating the spatial relationship of individual trees in a given plantation offer a greater opportunity to study the development of a stand of trees over time (Pielou 1977, Newnham and Smith 1964, Daniels et al. 1979, Wensel 1975). One aspect of tree growth and stand development that has not been adequately addressed is the effect of size and spatial distribution of vacancies, created by mortality and/or thinning on the growth and development of the remaining trees in a forest stand over time.

One of the major disadvantages of modeling the effects of vacancies on individual tree growth has been the use of single-equation estimation procedures employed to estimate the structural coefficients of a system of equations (Theil 1971, Sullivan and Clutter 1972). In such models it is assumed that the variables simultaneously satisfy certain assumptions pertaining to the standard linear model which assumes homoscedasticity (equal variances among regressors) and that the explanatory variables (i.e. endogenous and exogenous variables) are independent of the residual errors. The use of a single-equation estimation procedure in fitting individual equations to a set of data does not take into account the interrelationship between regressors as well as the interaction among equations (Hock 1958). Likewise, the procedure does not consider the joint distribution of the endogenous variables among equations or the properties associated with the disturbances. It also disregards any restrictions imposed by other equations on certain variables or coefficients in the system of equations (Haavelmo 1943).

Thus, studies similar to those aimed at evaluating the growth and development of stands over time should not be confined to individual equations but rather be extended to a system of equations (Zellner and Theil 1962). The aim of such a system is to describe a set of endogenous variables in terms of a set of exogenous ones and disturbances. The exogenous variables which occur in a system are determined outside of the system in question. On the other hand, the endogenous variables are determined jointly and simultaneously by the exogenous variables and the disturbances generated by the structural equations in the systems (Theil 1971).

This approach to estimating the parameters of a system of equations has two main advantages over the single-equation estimation procedure: it allows restrictions to be placed on certain variables, and permits the interaction of variables between equations to be considered in the estimation procedure.

The use of simultaneous equations could lend itself readily to the development of simulation and forecasting models designed to quantify the effects of changes in the degree of competition and growing space on the growth of individual trees in a given forest stand.

Objectives

The objectives of this study were (a) to model the growth and development of individual slash pine trees over time and (b) to quantify the effects of competition and available growing space on the growth of individual trees.

As the demand for wood products increases and the land base suitable for growing wood biomass continues to decline, forest management practices have intensified. Silvicultural practices aimed at increasing the productivity per unit land area include use of genetically improved seedlings, fertilization, thinning, bedding, prescribed burning, and drainage. The assessment of silvicultural practices in terms of stand management is complicated by uncertainties in the growth and development of individual stands due to differences in mortality, site, climatic factors, etc. (Lembersky and Johnson 1975). Recent advances in computer technology and its application to forestry have led to the development of realistic growth and yield models capable of assisting forest managers in assessing the effects of various silvicultural treatments and stand conditions on the growth and development of individual stands over time (Nance et al. 1982, Allen and Duzan 1980, Harms 1982). Knowledge gained from these models has helped foresters in developing optimal management strategies to effectively achieve specific objectives (Allen and Duzan 1980).

To fully exploit the potential of a given site for timber production, there is a need to better understand how competition and available growing space affects tree growth (Harms and Langdon 1977). Two of the more important factors are stand density and site quality.

Site is defined by the Society of American Foresters (Ford-Robertson 1971, p. 242) as "an area considered in terms of its environment, particularly as this determines the type and quality of the vegetation the area can carry". Sites may be classified either qualitatively or quantitatively.

Stand density is defined as "a measure of the degree of crowding of trees within a given area". It is commonly measured as a ratio of crown length to tree height, crown diameter to DBH, or crown diameter to tree height (Ford-Robertson 1971, p. 255). In a given plantation, the initial spacing exerts a strong influence over the size and quality of individual trees making up the stand. Density is one of the most important characteristics a forest manager has available for controlling the development of a stand over time (Shepard 1974). Formerly, planting densities were selected on the basis of prior experience in stands grown on similar sites. Recently, stocking levels have been determined on the basis of the cost of establishment, interest rates, rotation length, product utilization and other economic factors affecting the production of wood.

A reliable tree growth model must be capable of predicting the growth response of individual trees over a wide array of planting densities for a given site. However, the biological potential of the various sites for timber production varies considerably throughout the range of the slash pine ecosystem (Boyce et al. 1975). Even within relatively small areas site productivity may vary due to microsite differences (Swindel and Squillace 1980).

Extensive research has shown that as stand density increases there is a progressive decrease in the average size and value of an individual

tree as well as a change in its morphological and physiological characteristics (Davis 1966, Jorgenson 1967, Harms and Langdon 1977). This phenomenon is attributed to the increased competition for available growing space, light, nutrients and water. As stand density increases the ability of some trees to compete with their neighboring trees diminishes. The effects of stand density on tree growth is more complex than a simple reduction in overall growth due to competition, but is a result of a combination of factors such as the initial planting density, genetic traits, disease and insect infestation, spatial distribution of mortality and climatic conditions (Davis 1966).

Competition between neighboring trees results in a natural selection of those trees having the best possible complement of genetic traits for a given site and stand condition (Wright 1976, Jorgenson 1967). Inferior trees are gradually suppressed and shaded out. The number and distribution of suppressed trees depends upon the initial spacing of the stand, early mortality, the age at which the tree canopy closes, and the quality of the site. In the past, forest managers tended to overplant in order to ensure an adequate stocking level at the end of the rotation due to uncertainties in the survival rate. The effects of spacing on the size of individual tree variables are discussed below.

Total Tree Height

It has long been assumed that height growth is independent of stand density (Bennett 1963, Lloyd and Jones 1982, Husch et al. 1982). Usually, the mean height of a stand increases with increased spacing

(Shepard 1974, Jorgensen 1967, Bennett 1971). The closer the spacing the greater will be the intensity of competition among neighboring trees. In such cases, the proportion of suppressed trees will increase.

Bennett (1963) found that slash pine planted on old-field sites (site index 65, base age 25) grew approximately three feet annually for the first 15 years. Height growth increased with site quality and decreased with stand age. In a later study, Bennett (1971) observed that height growth declines after age ten years for all sites and densities.

Bennett (1963) also showed that during a given year height growth increases at a decreasing rate. By the end of March approximately 25 percent of the height growth is completed. The remaining percent of the height growth is completed by the end of June.

Lloyd and Jones (1982) pointed out that a substantial bias can result in growth predictions based on density-height independent models developed from data which include an effect of density on tree heights. The authors suggest that stand density should be used in modeling individual tree heights in order to obtain a more realistic height growth function. If a significant height-density relationship exists in the data, the omission of this information may result in a specification error which in turn affects the estimation of the coefficients in the model. The magnitude and direction of this bias depend solely upon the height-density relationship present in the development of the site index curves. Standard regression techniques may be employed to test the significance of individual variables in a given model.

Lloyd and Jones (1982) concluded that any attempt to modify site index to reflect stand density would be inappropriate since site index, by definition, is independent of density.

Tree Diameter

In general, the increase in the mean stand diameter is inversely proportional to the planting density. The configuration of the spacing (square versus rectangular) is not correlated with diameter growth (Bennett 1963). Campbell and Mann (1974) demonstrated that the initial planting density has a stronger influence on diameters than site index. As spacing and site index increase, the mean diameter of the stand shifts to the right resulting in a distribution skewed to the left. The interrelationships between stand density, individual tree size, competition and other factors control the rate of tree movement to higher diameter classes over time and, thus, the shape of the diameter distribution.

Stand density does not have a significant effect on average stand diameter until the fifth growing season (at standard stocking levels); it intensifies thereafter (Bennett 1963). Several researchers have demonstrated that most of the differences in diameters occur during the first 12 to 20 years (Shepard 1974, Jorgensen 1967). At closer spacings, growth is reduced due to increase in competition between trees. The mean periodic annual diameter growth is directly related to stand density and decreases with stand age (Lohrey 1961, Shepard 1974). When the differences in diameter growth between spacings stabilize, competition is severe for all spacings. On poor sites, it takes longer for differences in diameter growth to stabilize over time (Lohrey 1961).

Cubic Foot Volumes

For a given species, volumes of individual trees and stands are directly related to age, site index, and planting density of which only spacing can be controlled by the forester. Maximum yields are realized in higher planting densities and sites. The increase in volume is proportional to stand density (Bennett 1963, Lohrey 1961, Campbell and Mann 1974). On sites below 50, spacing has little or no effect on yields because of the inefficiency of the trees in the stand to fully utilize the site (Bennett 1963). The percentage increase in yields per unit area due to an increase in site is greater for lower sites than for higher sites, but the actual volume increase is larger for the higher sites. Also, yields tend to be higher on old-field sites than on natural forest sites due to a reduction in competing understory vegetation and residual effects from past fertilization (Bennett 1963). On an individual tree basis, volume growth increases with an increase in spacing. The economic advantage of the wider spacings are due mainly to the increase in wood quality which yields higher monetary returns. However, total wood productivity of the wider spaced stand may decrease.

Bennett (1971) indicates that current annual volume increment of slash pine plantations culminates at age 12 years in stands with approximately 200 trees per acre over all sites greater than 50. Culmination occurs later for higher densities but seldom takes place beyond 25 years after planting. Knowledge of the change in the current annual increment is important because it provides the forester with a better understanding of stand growth over time, and a basis for

determining optimal rotation lengths. Culmination of volume growth does not coincide with peak periods of height and diameter growth because of differences in growth rates. For a given stand, the combination of height and diameter growth results in maximum volume growth per unit land area for that site and stand density.

Tree Form

The effect of competition and available growing space not only affect the size of individual trees but also the form of the main stem (Lohrey 1961, Horn 1961, Meng 1980). In a study with red pine (_Pinus resinosa_ Ait.) plantations Horn (1961) observed that stem form improved with competition and age while thinning impaired tree form of dominant and codominant trees. Meng (1980) found that there was a statistically significant change in stem form following heavy thinning (50 percent of the basal area removed) and fertilization (448 kg/ha) of black spruce (_Picea mariana_ (Mill.) B.S.P.) stands. The change in the shape of the tree bole is in response to a decrease in competition and an increase in the available growing space (Horn 1961, Husch et al. 1982).

The diminution in diameters along the main stem is known as taper and is the fundamental reason for the variation in cubic volumes of trees with the same diameter at breast height and total tree height (Husch et al. 1982, Grosenbaugh 1966). Stem form also varies with species, diameter, age, and the quality of the site. Because of the large variation in stem forms among trees, there rarely exists any resemblance to one of the most common geometric solids (Grosenbaugh 1966, Husch et al. 1982).

Forest mensurationists have sought to discover a simple two-variable function involving only a few parameters which could be used to specify the entire tree profile (Grosenbaugh 1966). For example, Demaerschalk (1972) developed a system in which taper equations can be derived from existing volume equations; both equations yield identical results for total cubic volume if some taper data are available. In a similar study, Bailey et al. (1982) developed taper equations for site-prepared slash pine plantations.

Grosenbaugh (1966) demonstrated that total height (THT), bole surface area (SUR), and volume (VOL) of a given tree are highly correlated with other variables that are affected by the size and form of the tree stem, such as quality or quantity of lumber, plywood, kraft pulp yields as well as specific gravity. Not only are heights, surface areas, and volumes useful in estimating various product yields, but comparisons of ratios involving the three variables THT, VOL, and SUR provide additional information on the form of an individual tree. Grosenbaugh (1966) introduced three measures of tree form based on the ratios of height, volume, and surface areas, namely:

Quadratic mean dia. (in.), $Q = 13.5406 \ (\text{VOL/THT})^{1/2}$,

Arithmetic mean dia. (in.), $A = 3.8197 \ \text{SUR/THT}$, and

(Quad. Mean)^2/Arith. mean (in.), $Q^2/A = 48.0000 \ \text{VOL/SUR}$.

The three means coincide if a tree is a cylinder (no taper) and become increasingly different in the presence of taper. The ratio

(Arith. Mean)^2/ (Quad. Mean)^2, $F = 0.0796 \ \text{SUR}^2/\text{THT*VOL}$

which will always be less than one, measures the inverse of the relative

variation in diameter along the main stem. Zero variation is indicated by a ratio of unity or when the tree is cylindrical. As relative variation increases, the ratio becomes increasingly smaller than unity.

Stand Mortality

One aspect of forest growth that has not been adequately addressed in the past pertains to the size, spatial distribution, and abundance of vacancies created by mortality in forest stands as well as the anticipated effect on individual tree growth (Bormann and Likens 1981).

If vacancies in a given stand are randomly or uniformly distributed in space, adjacent trees may utilize the additional growing space to accelerate their growth. The net result may be an increase in stand productivity because of the reduced competition among neighboring trees. If vacancies are aggregated, the added growing space may not be fully utilized by adjacent trees. In such cases a loss in stand yield may be realized which is proportional to the area occupied by the dead trees (Newnham and Smith 1964, Stauffer 1978). Jones (1977) demonstrated that the spatial distribution of residual trees within a given stand significantly contributed to differences in individual tree volumes as well as total stand volumes.

Tree mortality is extremely variable and difficult to predict (Jones 1977). There are two basic types of mortality that occur in forest stands: regular and irregular (Lee 1971). Regular mortality include trees that die due to competition for limited resources such as light, moisture, growing space and nutrients. Mortality caused by insect and disease infestations, as well as windfall and fire, is termed

irregular. Predictions for both types of mortality are cumbersome and highly variable no matter how much mortality data are available (Davis 1966, Mitchell 1969, Hamilton 1974).

Competition

During the development of a stand, the growth of some trees is accelerated, while in others growth is suppressed or nonexistent. When openings in the stand are created by some kind of nonpersistent disturbances, adjacent trees tend to utilize the vacancies to increase their dominance within the stand (Mitchell 1969). The rate of growth depends upon the vigor of the adjacent trees to successively compete for the additional growing space (Bormann and Likens 1981).

A problem arises when analyzing the growth of individual trees due to the difficulty of quantifying the level of competition among neighboring trees (Daniels 1976, Opie 1968, Zedaker 1982). Initially, various measures of stand density in the vicinity of a given tree were used as a measure of competition. Such measures include basal area, angle count, point density, distances to neighboring trees, crown diameter, and the number of primary and secondary competing trees (Bella 1971). The concept of "zone of influence" as a measure of crown competition among neighboring trees has also been advocated. It is defined as the area of crown overlap between neighboring trees (Bella 1971, Venzina 1963).

The basic assumptions of various competition indices are

a. The ability of a tree to compete for limited resources such as growing space and light can be represented by a circle or zone

surrounding the tree, with the radius being a function of tree size such as volume, bole surface area, crown radius, DBH and total tree height, and

b. The competition stress experienced by a tree is a function of the degree to which its influence zone is overlapped by those of its neighbors.

Several modifications have been made in defining the influence-zone, including the use of weighted variables to denote the relative importance of the influence area overlap of a given tree to compete with neighboring trees (Daniels 1976, Venzina 1963). To estimate the growth potential of individual trees, Ek and Monserud (1974) used a weighted competition index based on the ratio of the total height of the subject tree to the average height of its competing neighbors multiplied by the crown radius of the subject tree. Other approaches advocated as a measure of competition are based on the ratio of diameters between the subject tree and its competing neighbors. This approach assigns weights proportional to the size of trees while the effects between trees of similar size remain essentially unweighted (Daniels 1976). Basal area has also been used as a measure of competition among neighboring trees. The basic assumption is that competition among individuals increases proportionally with the basal area of the stand (Bella 1971). Minor (1951) showed a slightly curvilinear relationship between diameter at breast height (DBH) and crown diameter. Minor also demonstrated that the ratio of DBH to crown diameter remains relatively constant once the basal area of a stand is stabilized.

Distance Sampling

Competition among individual trees within a stand is also directly related to the spatial distribution and size of the vacancies created by mortality. Therefore, it is important to approximate as closely as possible the spatial distribution of a given tree population in order to make valid inferences. Most sampling techniques and analyses were developed under the assumption of randomly distributed plant populations (Clark and Evans 1954). This assumption may not be valid for many plant communities (Ashby 1935). If the distribution of a population departs significantly from the underlying assumption of randomness, the resulting statistics and the reliability of the inferences may be in serious error (Cole 1946).

Distance sampling is one of the methods employed to study spatial patterns of plant populations over a continuous surface. Basically, this method involves distance measurements either from randomly selected points and/or from randomly selected plants to their nearest neighbor. The empirical frequency distribution of such distances and the associated statistics such as means and variances may be used in specific cases to test the degree of departure from randomness. Since random spatial patterns of plants are not very common in the real world, distance sampling methods have been applied to study the degree of aggregation or clustering of plants in a given area.

Recent findings (Reich 1980) indicate that Hopkins' modified coefficient of aggregation is an accurate and efficient computer sampling technique capable of analyzing the spatial patterns of natural and planted pine stands.

Hopkins' coefficient of aggregation, A, is based on the ratio of the squared distances between randomly selected points and the nearest tree over the squared distances between randomly selected trees and their nearest neighbor:

$$A = \frac{\Sigma \ \omega_1}{\Sigma \ \omega_2}$$

where

A = Hopkins' coefficient of aggregation,

$\Sigma\omega_1$ = Sum of squared point-to-tree distances,

$\Sigma\omega_2$ = Sum of squared tree-to-tree distances.

Pielou (1977) introduced a modified form of Hopkins' index that is more sensitive in detecting nonrandom spatial patterns. Hopkins' modified coefficient of aggregation is defined as

$$A^* = \frac{A}{1 + A}$$

which has a beta-type distribution with mean 0.5 and variance $1/4(2n+1)$, where n is the number of point-to-tree and tree-to-tree distances in the sample. With a large sample size (n>50) Hopkins' index approaches a normal distribution. In such cases $Z = 2 \ (A^* - 0.5)*(2n +1)^{1/2}$ can be viewed as a standard normal variate. The null hypothesis of a random spatial distribution would be rejected when

$$|Z| > Z_{\alpha/2}$$

where $Z_{\alpha/2}$ is the critical Z-value for the standard normal distribution. The population is said to be aggregate if A^* is

significantly greater than 0.5 and uniformly distributed if A* is significantly less than 0.5.

System of Equations

Most growth simulation models include equations to predict DBH, total tree height, crown width, and volume. They also incorporate some measure of competition to quantify the degree of interaction between neighboring trees. The predictive equations are not independent. When they are combined together they form a system of equations with correlated errors. This interdependence among regressors violates the assumption of a single equation estimation procedure which assumes that no correlation exists among the explanatory variables.

The above discussion suggests that studies of single tree growth should not be confined to individual equations but rather be extended to a system of L equations commonly referred to as simultaneous equations.

In the development of any tree growth model the most important consideration is the form of the relationship describing the interaction of individual tree parameters with growth. It is sometimes argued that the theory related to forest growth should provide insight into the relationship that the individual equations take on (Fisher 1970). The problem thus is to decide which set of basic characteristics adequately relates individual tree growth to cubic volume, surface area, tree diameter, total tree height, competition and available growing space.

Over the years numerous techniques have been developed to estimate the parameters associated with a system of equations. Several of the more common estimation procedures include Limited-Information Maximum-

Likelihood (LIML) procedure, Full-Information Maximum-Likelihood (FIML) procedure, Zellner's seemingly unrelated least squares procedure, Two-Stage Least Squares (2SLS), and Three-Stage Least Squares (3SLS) to name but a few. The appropriate procedure to use in estimating the parameters of a system of simultaneous equations will depend upon the structure and characteristics of the equations making up the system.

For example, LIML takes into account the absence of certain variables from the lth structural equation but does not assume the absence of any variables from the remaining L-1 equations in the system when estimating the parameters of the lth equation. On the other hand, FIML handles restrictions on the coefficients of all L equations simultaneously (i.e. zero coefficients in the lth equation). The 2SLS procedure is similar to LIML while the 3SLS procedure is similar to FIML. The FIML and LIML procedures are rarely used because the estimation procedure involves solving a very complicated set of estimation equations (Theil 1971).

A recent contribution to the estimation of a set of coefficients for a system of simultaneous linear equations is the 3SLS procedure developed by Zellner and Theil (1962). This procedure yields estimates which are asymptotically at least as efficient as the 2SLS procedure (Madangly 1964). The 2SLS procedure is applied to each of the L structural equations one at a time but computes a joint asymptotic standard error for the system as a whole, while the 3SLS procedure (an extension of the 2SLS procedure) handles the unknown coefficients of all L structural equations at the same time.

STUDY PROCEDURES

Population

The data used in this study were provided by Container Corporation of America. The study area is located near Callahan, Florida, in Nassau County.

Slash pine seedlings were hand planted in February 1962 in twelve one-acre plots measuring 2.5x4 chains at six different spacings: 5x10, 7x10, 9x10, 11x10, 13x10, and 15x10 feet. Each plot was replicated twice and consisted of 14 evenly spaced rows. Each plot was surrounded by a buffer strip. Two spacing plots were replanted in February 1963 due to low planting survival (Figure 1).

Prior to planting, the study area was part of an improved pasture. The pasture was burned prior to planting in 1962. The site is somewhat poorly drained with a spodic horizon 1.2 to 2.0 meters in depth.

All trees were assessed for survival at two years of age and then approximately every two years thereafter, starting at six years of age. Tree measurements were collected on a subsample of trees from each plot. Sample data included the condition of the tree (healthy or dead), the relative location within the study area, diameter at breast height, total tree height, merchantable tree height to a four inch top, tree vigor (position in the canopy) and the number of primary and secondary competing trees.

19
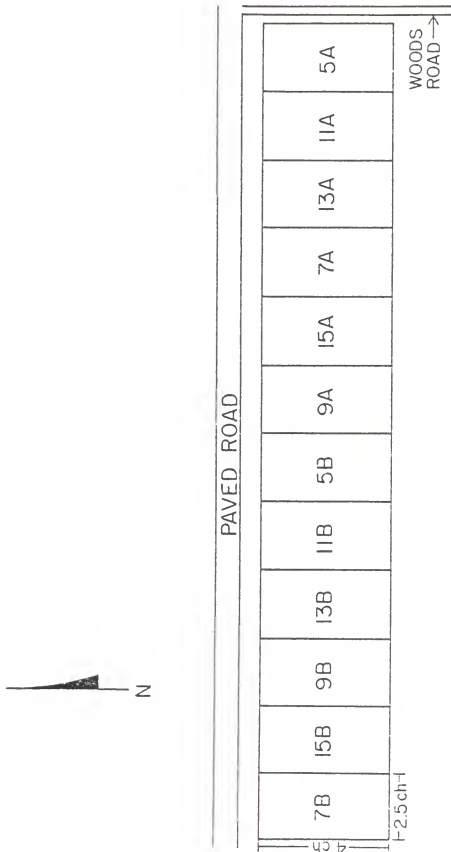
Figure 1. Slash pine spacing study, McArthur Tract, near Callahan, Florida, installed in February 1962.

Prior to the fifth set of measurements, sample trees were selected from two diagonal transects established in each plot. On each transect 14 trees, one from each row, were sampled (28 trees per plot). Commencing from the fifth set of measurements on, four rows were systematically selected from each plot and all trees within the selected rows were measured. The number of sample trees per row varied by spacing and degree of survival.

Spatial Analysis

Hopkins' modified coefficient of aggregation was used to analyze the spatial pattern of healthy and dead trees in the study area when the trees were four years old. Age four was selected for analysis because almost all mortality in the study area had already occurred by that time. In each plot, ten independent estimates of Hopkins' index $A*$ were obtained based on 150 point-to-tree and 150 tree-to-tree distances. A z-statistic was used to determine the degree of departure of the observed spatial distribution from the theoretical (random) one. Analysis of variance was employed to evaluate the magnitude of differences in the spatial pattern of healthy and dead trees among the six spacings. Mean separation was determined using Duncan's multiple range test ($\alpha = 0.05$).

Microsite Variation

Swindel and Squillace (1980) demonstrated the existence of microsite variation on the growth of individual trees. To examine this extraneous variation on tree growth over time, the effects of spacing and replications were removed from the survival, DBH, total tree height, cubic volume, and surface area data for each time period. Spacing effects (S_i) were estimated as the difference between the spacing means (SP_i) and the grand mean (GM); replication effects (R_j) were estimated as the difference between the replication means (REP_j) and the grand mean. Microsite was estimated by subtracting the effects of spacing and replications from the individual plot means:

$$\bar{Y}'_{ij} = \bar{Y}_{ij} - S_i - R_j$$

$$S_i = SP_i - GM$$

$$R_j = REP_j - GM$$

where

$\bar{Y}'_{ij}$ = corrected plot mean for spacing i and replication j,

$\bar{Y}_{ij}$ = uncorrected plot mean for spacing i and replication j,

$SP_i \;\; = \dfrac{1}{2} \sum\limits_{j=1}^{2} \bar{Y}_{ij}$ spacing mean,

$REP_j = \dfrac{1}{6} \sum\limits_{i=1}^{6} \bar{Y}_{ij}$ replication mean,

$GM \;\; = \dfrac{1}{12} \sum\limits_{i=1}^{6} \sum\limits_{j=1}^{2} \bar{Y}_{ij}$ grand mean,

$S_i \;\; =$ spacing effect,

$R_j \;\; =$ replication effect.

The effect of microsite on survival and individual tree size was analyzed by (a) comparing the corrected plot means at each age, and (b) comparing corrected plot means over time for a given plot.

Effects of Spacing on Individual Tree Growth

To estimate the effects of spacing on survival, DBH, total tree height, cubic volume and surface area, individual plot means ($\bar{Y}_{ij}$) were corrected for microsite and replication:

$$\bar{Y}_i^* = \bar{Y}_{ij} - M_{ij} - R_j$$

where

$M_{ij} = \bar{Y}_{ij}' - GM,$

$\bar{Y}_i^* =$ plot mean corrected for microsite and replication,

$M_{ij} =$ microsite effect.

Plot means for survival, DBH, total height, cubic volume, and surface area corrected for microsite and replication effects were regressed on spacing by age using the standard linear model:

$$\bar{Y}_i^* = \beta_o + \beta_1 \text{ (spacing)} + \varepsilon_i.$$

The significance of spacing on survival and individual tree size was tested using regression analysis ($\alpha = 0.05$).

Volume and Surface Area Estimation

A sample of 150 standing trees were selected at random for dendrometry measurements in the Spring of 1983. Approximately 12 trees were selected from each spacing x replication plot. Heights and corresponding upper stem diameters of standing sample trees were measured with a magnifying Barr and Stroud optical dendrometer, Model FP-15. Measurements were taken at stump height (0.5 feet above ground), breast height (4.5 feet above ground), and various other points along the tree stem. This procedure divides each standing tree into segments that are relatively uniform with respect to taper, quality, and defects. Diameters at breast height were also measured with a diameter tape; bark thickness at breast height was measured with a bark guage.

Cubic foot volumes, bole surface area and heights inside and outside bark for each sample tree were computed and accumulated over all measured tree segments. Product yield estimates were computed with Grosenbaugh's (1974) STX computer program. Regression equations were developed to predict total cubic foot volumes (VOL) and bole surface areas (SUR), outside bark, of individual trees as a function of diameter at breast height (DBH) and total tree height (THT) using the functional forms:

$$VOL = \alpha \; DBH^{\beta} \; THT^{\gamma}$$

$$SUR = \alpha' DBH^{\beta'} THT^{\gamma'}$$

where $\alpha(\alpha')$, $\beta(\beta')$, and $\gamma(\gamma')$ are regression coefficients estimated from the sample dendrometry data obtained from each spacing. The two regression equations were fitted using a standard linear model after a logrithmic transformation.

The regression equations were used to estimate individual tree volumes and surface areas at 6, 8, 10, 12, 14, 16, 19, and 21 years after planting for all sample trees used in the analysis.

Tree Form

Using the volume and surface area equations described in the previous section, it is possible to express Grosenbaugh's (1966) four ratios of stem form in terms of diameter at breast height and total tree height by substituting the volume and surface area equations into the appropriate ratio. The four ratios will have the same functional form as the volume and surface area equations. Since trees can assume an infinite number of shapes, the coefficients in the above equations would tend to vary from tree to tree. Thus, equations developed for a given population can only predict the average change in tree form, cubic volume and surface area with respect to changes in diameters and heights of the same population.

Data Interpolation

The proposed growth model assumes a constant time interval (i.e. two years) between current and lagged exogenous and endogenous variables. To satisfy this constraint, three interpolation procedures (linear, Lagrangian, and quadratic) were examined to select an appropriate one to estimate DBH and total tree height at ages 4, 18, and 20 years after planting. The three interpolation procedures are described below

Linear Interpolation

If DBH and total tree height (y) are functions of the age of a tree (x), and y_0 and y_1 are known functional values corresponding to x_0 and x_1, respectively, the functional value of y can be approximated in a linear manner by

$$y = \frac{(y_1 - y_0)\, x + x_1 y_0 - x_0 y_1}{x_1 - x_0} \qquad \text{where, } x_0 < x < x_1 \ .$$

Lagrangian Interpolation

Given any three points on a plane (x_0, y_0), (x_1, y_1) and (x_2, y_2), it is possible to evaluate the Lagrangian interpolation of a second degree polynomial $P_2(x)$ which passes through the three points. The functional value of x, denoted by $P_2(x)$, can be approximated by

$$P_2(x) = \sum_{i=0}^{2} L_i(x) y_i$$

where

$$L_i(x) = \prod_{\substack{j=0 \\ i \neq j}}^{2} \frac{(x - x_j)}{(x_i - x_j)} \ .$$

Quadratic Interpolation

The quadratic procedure assumes no functional relationship between x and y. Given two values y_0 and y_1, the value of y for a given x can be approximated by

$$y = \left[\prod_{i=0}^{1} y_i \right]^{1/2} \qquad \text{where, } y_0 < y < y_1.$$

Evaluation of the three interpolation procedures indicated the linear method diameters and total heights during the juvinile growth phase and underestimates during the latter years. The quadratic procedure was not considered because it estimates a single geometric

mean regardless of whether X is much closer to X_2 than X_1. As a result, the Lagrangian procedure was used to estimate DBH and total tree height at ages 4, 18, and 20 years. The interpolated data was then used to estimate total cubic volume and surface area.

Growth Function

The cumulative growth of an individual tree over time resembles an S — or sigmoid — shaped curve. Although the exact form of the cumulative growth curve changes with the dimensions of the tree (diameter, height, volume and surface area) the cumulative growth curve has certain properties that hold for all dimensions of a tree (Husch et al. 1982).

Grosenbaugh (1965) investigated numerous sigmoid and nonlinear functions capable of generating a wide variety of sigmoid and other monotonically changing shapes consistent with the growth of individual trees over time. Several nonlinear growth functions were examined to select one that is intrinsically linear with respect to the parameters in the growth model and that are consistent with the growth behavior of individual trees subjected to various competitive stresses. A few of the relationships examined by Grosenbaugh include the Johnson-Schumacher, Gompertz, Gauss, von Bertalanffy, Verhulst and the Pearl-Reed growth functions. Principal features and asymptotic properties of the above growth functions are discussed by Grosenbaugh (1965).

The growth function selected to model tree growth in this study was that of Verhulst which has the following form:

$$Y = A \ (1 + C \exp \ (- \ BX))^{-1}$$

where

Y = dependent variable,

X = independent variable,

A = asymptotic maximum of Y, (YMAX),

C,B = regression coefficients.

The Verhulst growth function was modified to include more than one independent variable in the growth model and to express individual tree growth on a percentage basis. The latter allows for comparison of the effects of competition and relative growing space on diameters, heights, volumes and surface areas over time. The modified Verhulst function has the following form:

$$Y* = (1 + \exp(Y'))^{-1}$$

where,

$Y* = Y/A$ or $Y/YMAX$, $0 < Y* < 1$, and

$$Y' = B_0 + B_1X_1 + B_2X_2 + \ldots + B_kX_k + \varepsilon \ .$$

Taking the first derivative of $Y*$ with respect to Y' yields

$$\frac{dY*}{dY'} = - \ Y* \ (1 - Y*) < 0.$$

The second derivative is given by

$$\frac{d^2Y*}{dY'^2} = (1 - 2Y*)(1 - Y*) \ Y* \gtrless 0.$$

The asymptotic properties of the modified Verhulst function are

$$\lim_{Y' \to -\infty} Y^* = 1$$

$$\lim_{Y' \to \infty} Y^* = 0.$$

To utilize the modified Verhulst growth function in modeling individual tree growth over time, the value of YMAX must be defined in terms of the dependent variables being modeled (diameter, height, volume and surface area). Since YMAX by definition is the maximum size an individual tree can attain, YMAX was equated to the site productivity of the stand. Maximum values for DMAX, HMAX, VMAX and SMAX were defined as follows:

HMAX = site index base age 50,

DMAX = maximum tree diameter for a given site index (HMAX),

VMAX = maximum cubic foot volume; f(DMAX, HMAX), and

SMAX = maximum bole surface area; f(DMAX, HMAX).

Model Development

When one performs several statistical tests to identify the structural form of a regression equation and uses the data to estimate the parameters of the model, certain problems arise which can affect the sampling distribution of the estimators (Theil 1971, Snee 1977). This problem is overcome when the identification and estimation procedures are based on independent data sets. It is usually recommended that the

data set be split into two groups; the first set is used to identify the structural form of the equations, while the second set provides estimates of the coefficients of the model.

The main advantage of data splitting is that one can obtain an independent estimate of the prediction accuracy of the model. The disadvantage of data splitting is that the variance of the estimated coefficients in the model increases. Snee (1977) presented a simple formula to approximate the percent increase in the variance when the data are split into two groups of equal size:

$$\text{Percent Increase} = \left[\left[\frac{(2n-1)}{(n-1)}\right]^{1/2} - 1\right] 100$$

where n is the size of the estimation and prediction data sets. With a sample size n = 30 the increase in the variance is approximately 42.6 percent and rapidly approaches 41.4 percent as the sample size goes to infinity. Because of this increase in the variance most researchers use the same data set to identify and estimate the structural form of the equations.

In this study the sample data were sorted by age, spacing, and replication and numbered from 1 to 5199. The data were sorted in this manner to increase the correlation among the independent variables over the six planting densities and nine time periods. Subsequently, the data were divided into two groups of equal size (N1 = 2600, N2 = 2599). All odd observations were included in data set one; even observations were included in the second data set. The first data set (N1) was used to identify the structural form of the growth equations using the OLS procedure, while the second data set (N2) was used to estimate the coefficients of the system of equations using the 3SLS procedure.

Field Data Screening

REX (Grosenbaugh 1967), a FORTRAN-IV computer program, was used for combinatorial screening of variables used in the growth model for this study. This approach explores all possible linear combinations of variables to find the "best" combination of independent variables that best explains the variation of the dependent variables. REX uses the mean squared residual, MSE (or variance of the regression estimate), as an indicator of a "best fit" since the MSE does not change monotonically with the number of independent variables in the equation. As opposed to other procedures such as stepwise, the selection of the "best" fitting model by REX is independent of the steps used in the exploratory analysis and provides unbiased estimates of the coefficients, but tends to underestimate the variance of prediction.

Fourteen variables were screened through REX for the diameter and height equations; thirteen variables were screened for the volume and surface area equations (Table 1). The final equations for diameter, total height, cubic volume and surface area had the following functional form:

$$D_1 = \beta_{10} + \beta_{11}D_{-1} + \beta_{12}H_{-1} + \beta_{13}PAR + \beta_{14}PAR^2 + \beta_{15}R2 + \beta_{16}A + \beta_{18}/A^2 + \beta_{110}COMP$$
$$+ \beta_{112}DMS + \varepsilon_1$$

$$H_2 = \beta_{20} + \beta_{21}D_{-1} + \beta_{22}H_{-1} + \beta_{23}PAR + \beta_{24}PAR^2 + \beta_{25}R2 + \beta_{26}A + \beta_{27}A^2 + \beta_{29}R1P$$
$$+ \beta_{211}COMP/SP + \varepsilon_2$$

$$V_3 = \Gamma_{31}D + \Gamma_{32}H + \beta_{30} + \beta_{33}PAR + \beta_{34}PAR^2 + \beta_{35}R2 + \beta_{36}A + \beta_{38}/A^2 +$$
$$\beta_{39}R1P + \beta_{311}COMP/SP + \varepsilon_3$$

$$S_4 = \Gamma_{41}D + \Gamma_{42}H + \beta_{40} + \beta_{43}PAR + \beta_{44}PAR2 + \beta_{45}R2 + \beta_{46}A + \beta_{47}A^2 +$$

$$\beta_{49}R1P + \beta_{411}COMP/SP + \varepsilon_4$$

where

$D_1 = Ln\left(\frac{1}{D*} - 1\right)$

$D_1^* = DBH/DMAX$

DMAX = maximum DBH at 50 years of age

$D_{-1} = D_1$ lagged one period (two years)

$H_2 = Ln\left(\frac{1}{H*} - 1\right)$

$H_2^* = THT/HMAX$

HMAX = site index, base index 50 years

$H_{-1} = H_2$ lagged one period

$V_3 = ln\left(\frac{1}{V*} - 1\right)$

$V_3^* = VOL/VMAX$

VMAX = maximum cubic volume at age 50 years for a given DMAX
        and HMAX

$S_4 = Ln\left(\frac{1}{S*} - 1\right)$

$S_4^* = SUR/SMAX$

SMAX = maximum bole surface area at 50 years of age for a given
        DMAX and HMAX

PAR = relative growing space = $\frac{SP}{5(kL+1)}$

KL = number of nearest neighbors lagged one time period,
    KL = 0,1,2

SP = spacing within rows (feet)

A = tree age (years)

A_{-1} = tree age lagged one period

$R2 = A_{-1}/A$

$R1P = (1 - R2) * PAR$

$COMP = \sum_{k=1}^{2} DBH_k/(2 * DBH)$

DBH_k = DBH of neighboring trees within rows (inches)

DBH = DBH of subject tree (inches)

DMS = diameter microsite (inches)

ε_ℓ = error term for the lth structural equation

Ln = natural logarithm

Γ_{ij}, β_{ij} = parameters of the endogenous and exogenous variables, respectively.

The equations for diameter, total height, cubic volume and surface area were fitted using REX to obtain OLS estimates of the coefficients in the model. The coefficients were estimated assuming that diameter and height were exogenous variables in the cubic volume and surface area equations and that the disturbances among equations were uncorrelated. The four equations were also fitted using SAS/SYSREG (Helwig and Council 1979) procedure to obtain 3SLS estimates. This procedure assumes a correlation of residuals among the four equations along with the presence of endogenous variables as explanatory ones. Following is a detailed description of the 3SLS procedure.

Table 1. Variables used to identify the "best" combination of explanatory variables.

Independent	Equation			
Variable	DBH(D)	THT(H)	VOL(V)	SUR(S)
DBH(D)	N.U. [1]	N.U.	A	A
Total tree height (H)	N.U.	N.U.	A	A
DBH Lagged (D_{-1})	A [2]	A	N.U.	N.U.
Total height lagged (H_{-1})	A	A	N.U.	N.U.
PAR	G	G	G	G
PAR **2	G	G	G	G
COMP *2	G [3]	G	G	G
COMP	S	S	S	S
COMP/SP	S	S	S	S
Alag/Age	G	G	G	G
Age	G	G	G	G
1/Age	S	S	S	S
Age **2	G	G	G	G
1/Age **2	S	S	S	S
(1-Alag/Age)*PAR	G	G	G	G
DBH Microsite	G	N.U.	N.U.	N.U.
THT Microsite	N.U.	G	N.U.	N.U.

[1] Not used in the screening process.

[2] Variable was automatically included in the model and was not subject to the screening process.

[3] G = group; S = set; members of sets within a group are treated as nonfixed, and are included in regressions only when called for by specific combinatorial rules. For example, the competition group contains three sets; only one set will be included in the model if the group is selected in the screening process.

Description of the System of Equations

The system of equations described in the previous section contains L linear stochastic structural equations consisting of L jointly dependent variables and K predetermined variables. It is assumed that

(a) the system can be solved for the jointly dependent variables and

(b) the disturbances of the structural equations have zero mean and

(c) the disturbances are serially independent and homoscedastic. The latter implies that their variances and contemporaneous covariances are finite and constant over time. A test for cross-sectional and serial correlation will be discussed in a later section as well as the appropriate estimation procedure.

If there are n observations then any structural equation, say the lth equation, can be written in the following form for all n observations combined:

$$y_\ell = Y_\ell \gamma_\ell + X_\ell \beta_\ell + \varepsilon_\ell$$

where y_ℓ is a nxl column vector of observations for one of the L jointly dependent variables in the system; Y_ℓ is a nxL matrix of values taken on by the explanatory dependent variables in the ℓth equation; γ_ℓ is the corresponding Lxl coefficient vector; X_ℓ is a nxK matrix of values taken on by the exogenous variables; β_ℓ is the corresponding Kxl coefficient vector; and ε_ℓ is a nxl column vector of structural disturbances. Using matrix notation the system as a whole may be written as follows:

$$Y\Gamma + X\beta = E$$

where Y is a nxL matrix of jointly dependent variables; Γ is a LxL

matrix of parameters; X is a nxK matrix of predetermined variables; β is a KxL matrix of parameters; and E is a nxL matrix of disturbances. Since there are as many equations as jointly dependent variables, the matrix Γ is square and nonsingular by definition. Thus, the system may be solved for Y by

$$Y = -X\beta\Gamma^{-1} + E\Gamma^{-1}.$$

This system of equations is referred to as the reduced form. It describes the endogenous variables in terms of the current and lagged independent variables, lagged dependent variables and disturbances. The reduced form coefficients can be estimated indirectly from the structural equations as shown above or directly by the reduced form:

$$Y = X\tau + U$$

where $\tau = -\beta\Gamma^{-1}$ and $U = E\Gamma^{-1}$. However, if the coefficients of the reduced form equations are estimated directly, it is impossible to obtain estimates of the coefficients associated with the structural equations. The latter approach was used by Sullivan and Clutter (1972) to estimate the coefficients of a simultaneous growth and yield model for loblolly pine stands. Since the reduced form equations do not contain current endogenous variables, as explanatory variables, the L reduced form equations can be considered unrelated even though they still have correlated errors among equations. In such cases Zellner's seemingly unrelated least squares procedure would be the appropriate one to use in estimating the coefficients of the reduced form equations (Theil 1971).

Three-stage Least-Squares Procedure

The 3SLS procedure uses the results of the 2SLS procedure to estimate the coefficients of the entire system simultaneously.

The matrix $E(Y_\ell)$ is a submatrix of the matrix $E(Y)$ which corresponds to the jointly dependent variables that occur on the right hand side of the equation. The ℓth structural equation may then be expressed in the following form:

$$y_\ell = \left[E(Y_\ell) - Y_\ell \right] \begin{bmatrix} \gamma_\ell \\ \beta_\ell \end{bmatrix} + \varepsilon_\ell + \left[Y_\ell - E(Y_\ell) \right]$$

The first stage of the 3SLS procedure yields an ordinary least squares (OLS) estimate of $E(Y_\ell)$ which is given by

$$y_\ell = \left[Y_\ell - U_\ell : X_\ell \right] + \varepsilon_\ell + U_+ \; \delta_\ell$$

where

$$\delta_\ell = \left[\gamma_\ell : \beta_\ell \right]$$

$$U_\ell = M \; Y_\ell$$

$$M = I - X(X'X)^{-1}X'$$

$$E(Y_\ell) = X(X'X)^{-1} X'Y_\ell - MY_\ell \; .$$

In the second stage OLS is used to provide an estimate of the coefficient vector $\hat{\delta}_\ell$. Zellner and Theil (1962) and Theil (1971) have shown that the 2SLS estimate of δ_ℓ is given by

$$d_\ell = \left[Z'_\ell \; X(X'X)^{-1} \; X'Z_\ell \right]^{-1} Z'_\ell \; X(X'X)^{-1}X' \; Y_\ell$$

where $Z_\ell = \begin{bmatrix} Y_\ell - U_\ell : X_\ell \end{bmatrix}$. The same authors discuss in detail the estimation procedure in the two cited references. An estimate of the variance-covariance matrix S_ℓ is given by

$$S_{\ell\ell} \begin{bmatrix} Z_\ell' \, X(X'X)^{-1} X' Z_\ell \end{bmatrix}$$

from which asymptotic standard errors can be computed by taking the square roots of the diagonal elements. The scalar $S_{\ell\ell}$ is an estimate of the variance $\sigma_{\ell\ell}$ which is defined as the mean square of the associated residuals.

$$S_{\ell\ell} = \frac{1}{n} (Y_\ell - Z_\ell \, d_\ell) ' (Y_\ell - Z_\ell \, d_\ell)$$

To obtain the third stage estimates, the L structural equations, for all n observations, are stacked in the following manner:

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_L \end{bmatrix} = \begin{bmatrix} Z_1 & 0 & . & . & . & 0 \\ 0 & Z_2 & . & . & . & 0 \\ \cdot & \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot \cdot & \\ 0 & 0 & . & . \, . & Z_L \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \cdot \\ \cdot \\ \cdot \\ \delta_L \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_L \end{bmatrix}$$

or simply

$$y = Z\delta + E .$$

Pre-multiplying the above equation by the transpose of a LnxKL Kronecker product of the form:

$$I \otimes X' = \begin{bmatrix} X' & 0 & . & . & . & 0 \\ 0 & X' & . & . & . & 0 \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ 0 & 0 & . & . & . & X' \end{bmatrix}$$

where X' is an Kxn matrix of explanatory variables yields

$$(I \otimes X') \, y = (I \otimes X') \, Z\delta + (I \otimes X')E$$

where y is a Lnxl vector of endogenous variables.

Applying the Generalized Least Squares (GLS) procedure to the above system of equation leads to the following estimate of δ:

$$\hat{\delta} = (Z'[\Sigma^{-1} \otimes X(X'X)^{-1}X']Z)^{-1} \, Z'[\Sigma^{-1} \otimes X(X'X)^{-1}X']y.$$

If the variance-covariance matrix Σ is replaced by S = (s_{jj}), the matrix of mean squares and products of the 2SLS residuals leads to:

$$\hat{\delta} = (Z'[S^{-1} \otimes X(X'X)^{-1} \, X']Z)^{-1} \, Z'[S^{-1} \otimes X(X'X)^{-1}X']y$$

the three-stage least squares (3SLS) estimator.

Identification of the System of Equations

To obtain an efficient estimate of the coefficients of the structural system of equations and an estimate of the variance-covariance matrix, the sytem of equations must satisfy an order condition of identifiability (Theil 1971). A necessary condition for

the existence of the 3SLS estimator is that $K \geq N_\ell$, where K is the total number of predetermined variables in the system and N_ℓ is the number of parameters to be estimated in the ℓth structural equation. Three possible conditions exist $K \gtrless N$. If $K < N$, the ℓth equation is said to be under identified and the 2SLS estimator does not exist. This implies that at least one row or column of the variance-covariance matrix $\ddagger$ cannot be estimated. In other words the parameters of the ℓth equation are not estimable because of a linear combination of the other equations that contains only variables which do not occur in the ℓth equation. In such a situation, Zellner and Theil (1962) recommend the elimination of all under identifiable equations from the system and confinement of the 3SLS procedure only to identifiable equations.

If $K = N$ the ℓth equation is said to be just identified. If all the equations in the system are just identified or if the variance-covariance matrix $\ddagger$ is diagonal, the 3SLS estimator provides no gain over the 2SLS procedure (Theil 1971, Zellner and Theil 1962).

The 3SLS procedure provides an asymptotic gain over 2SLS when the structural disturbances have non-zero contemporaneous covariances and at least some of the equations are overidentified, $K > N$ (Zellner and Theil 1962). When the above conditions hold, the asymptotic covariance matrix of the 2SLS estimator d_ℓ will exceed that of the corresponding subvector of $\hat{\delta}$ by a positive semidefinite matrix. The asymptotic superiority of the 3SLS procedure is due to the optimality of the GLS procedure which underlies the joint estimation procedure (Theil 1971, Madangly 1964, Zellner and Theil 1962).

Contemporaneous Correlations

If the disturbances of the growth model are correlated across equations, a systems approach is required to assure asymptotically efficient parameter estimates. If the assumptions of the standard linear model hold for all equations, the elements of the disturbance vector ε have zero mean but with unequal variances for each equation in the system. In general, there are three types of disturbances associated with a system of simultaneous equations. The first $E(\varepsilon_\alpha, \varepsilon_\eta)$ and $E(\varepsilon_{\alpha'}, \varepsilon_{\eta'})$, for $\alpha \neq \eta$ concerns disturbances of different years but the same equation (serial correlation). In the standard linear model these covariances are assumed to be zero. The second type of covariance $E(\varepsilon_\alpha, \varepsilon_{\eta'})$ for $\alpha \neq \eta$ pertains to the disturbances of both different years and different equations. It is also assumed that these covariances are equal to zero. The third type of covariance $E(\varepsilon_\alpha, \varepsilon_{\alpha'})$ deals with disturbances of different equations but of the same year. This type of covariance is referred to as the contemporaneous covariance.

The disturbances associated with the four structural equations, diameter, total height, cubic volume and surface area are assumed to be correlated. There are several reasons for suggesting the contemporaneous nature of the disturbances. If two variables are correlated with a third common variable, they will also appear to be correlated with each other. Since both tree height and tree diameter are correlated with age, height also appears to be correlated with diameters (Husch et al. 1978). Since tree diameters and tree heights are commonly used to predict volumes and surface areas, it would be natural to assume that volumes and surface area are correlated with diameters and heights as well as with age.

The contemporaneous covariance matrix for a given observation has the following form:

$$
E\,(\varepsilon_{\alpha},\varepsilon_{\alpha'}) = \ddagger =
\begin{bmatrix}
\sigma_{11} & \sigma_{12} & \cdot & \cdot & \cdot & \sigma_{1L} \\
\sigma_{21} & \sigma_{22} & \cdot & \cdot & \cdot & \sigma_{2L} \\
\cdot & \cdot & \cdot & & & \cdot \\
\cdot & \cdot & & \cdot & & \cdot \\
\cdot & \cdot & & & \cdot & \cdot \\
\sigma_{L1} & \sigma_{L2} & \cdot & \cdot & \cdot & \sigma_{LL}
\end{bmatrix} \qquad .
$$

The diagonal elements σ_{ii} are the variances associated with the structural equations and the off-diagonal elements σ_{ij} are the contemporaneous covariances in the αth time period. The condition that $(\varepsilon_{ij} = \varepsilon_{ji})$ and the independence of the observations over time is an extension of the homoscedasticity constraint on the standard linear model. It also indicates that the disturbances are random variables from a multivariate population with zero mean and constant covariance matrix. The disturbance vector ε implies that the covariance matrix is block diagonal with n-diagonal submatrices all equal to $\ddagger$.

$$
V(\varepsilon_{\ell}) =
\begin{bmatrix}
\ddagger_1 & 0 & \cdot & \cdot & \cdot & 0 \\
0 & \ddagger_2 & \cdot & \cdot & \cdot & 0 \\
\cdot & \cdot & \cdot & & & \cdot \\
\cdot & \cdot & & \cdot & & \cdot \\
\cdot & \cdot & & & \cdot & \cdot \\
0 & 0 & \cdot & \cdot & \cdot & \ddagger_n
\end{bmatrix} \qquad .
$$

The off-diagonal submatrices are all zero since their elements are of the first $E(\varepsilon_\alpha, \varepsilon_\eta)$, $E(\varepsilon_{\alpha'}, \varepsilon_{\eta'})$, and second type $E(\varepsilon_\alpha, \varepsilon_{\eta'})$ for $\alpha \neq \eta$. The inverse of $V(\varepsilon_\ell)$ is also block diagonal with $\overset{\dagger}{\downarrow}{}_\ell^{-1}$ in the diagonal blocks. Estimates of the parameters of the lth equation in the system are obtained by the use of an Aitkens' estimator:

$$\hat{\beta}_\ell = \left(X'\overset{\dagger}{\downarrow}{}_\ell^{-1}X\right)^{-1}X'\overset{\dagger}{\downarrow}{}_\ell^{-1}y_\ell$$

where $\hat{\beta}_\ell$ is the coefficient vector for the lth equation; X is a n x K matrix of explanatory variables; and y_ℓ is an n x 1 vector of dependent observations for the lth equation. The difficulty in applying the Aitken estimator is that the matrix $\overset{\dagger}{\downarrow}$ is usually unknown and must be estimated from the sample data. However, the variance-covariance matrix $\overset{\dagger}{\downarrow}$ under 3SLS will be misspecified if a cross sectional and/or time series correlation exists among the equations in the system.

Cross Sectional Time Series Analysis

Data obtained by repeated measurements on an individual will contain correlated errors (Sullivan and Clutter 1972). Measurements taken closer together in time are more highly correlated than measurements taken further apart in time. Furthermore, unequal variances may be present in the data (McHugh and Wall 1962). In practice serial correlations and heterogeneous errors are frequently ignored and parameter estimates are obtained by ordinary least squares (OLS). The OLS estimates of the coefficients are unbiased but have inflated errors due to a specification error. Also, the estimate of the residual mean

squared error is biased downward (Sullivan and Clutter 1972). The presence of non-zero covariances and non-homogeneous variances provide a basis for considering the inclusion of this additional information into the estimation procedure to obtain a more efficient estimate of the error components of the model (McHugh and Wall 1962).

Consider the following cross-sectional time series model. Suppose N individual trees, $n = 1, 2, \ldots, N$ have been sampled over T periods of time, $t = 1, 2, \ldots, T$. The variable to be explained is denoted y_{nt}. The dependent variable is explained by K truly exogenous variables, and L lagged values of the dependent variable. The model may be expressed in the following manner:

$$y = Y_\theta \, \Gamma + X\beta + \varepsilon$$

where

$$y = \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_N \end{bmatrix} = \begin{bmatrix} y_{11} \\ \cdot \\ \cdot \\ y_{1T} \\ \cdot \\ \cdot \\ \cdot \\ y_{N1} \\ y_{NT} \end{bmatrix}$$

an NT x 1 vector;

$$X = \begin{bmatrix} x_{11}^1 & \cdot & \cdot & \cdot & x_{11}^K \\ \cdot & \cdot & & & \cdot \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ x_{NT}^1 & \cdot & \cdot & \cdot & x_{NT}^K \end{bmatrix}$$

an NT x K matrix of exogenous variables;

$$y_\theta = \begin{bmatrix} y^1_{1-1} & \cdot & \cdot & \cdot & y^L_{1-\theta} \\ \cdot & & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot \\ \cdot & & & & \cdot \cdot \\ y^1_{NT-1} & \cdot & \cdot & \cdot & y^L_{NT-\theta} \end{bmatrix}$$

an NT x L matrix of lagged values of the dependent variable;

$$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_N \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} \\ \cdot \\ \cdot \\ \varepsilon_{1T} \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_{N1} \\ \cdot \\ \cdot \\ \varepsilon_{NT} \end{bmatrix}$$

an NT x 1 vector of residuals. Also, β is a K x 1 vector of constant coefficients of the exogenous variables, and Γ is a L x 1 vector of coefficients for the lagged endogenous variables. The model can be simplified by letting $Z = [X: Y_\theta]$ an NT x K + θ matrix, and $\delta = [\beta:\Gamma]$ a K + θ x 1 vector of coefficients:

$$y = Z\delta + \varepsilon.$$

Rao (1959) considered the estimation of the parameters of a linear model for data containing correlated errors and derived estimators requiring an independent estimate of the variance-covariance matrix. Elston and Grizzle (1962) compared Rao's GLS procedure with that of OLS and suggested that OLS might in some cases provide unbiased estimates

because of its simplicity. Sullivan and Clutter (1972) suggested that OLS should not be ignored in the analysis of growth and yield data with correlated errors because of its simplicity and when parameter estimates rather than confidence intervals are desired for the parameters and predictions.

A large proportion of the theory relating to the growth and yield of individual trees is aimed at explaining the effects of various stand factors (i.e. stand density, age, and competition) on tree growth over time. However, the majority of the growth models currently in use have frequently been developed from data covering a large cross-section of stand conditions within a given population and the specification of the model is based on the overall response observed in the aggregated sample data. For example, the growth of an individual tree is dependent upon the initial planting density, level of competition, the age of the tree, and productivity of the site. Thus, for a given site and initial planting density, a theoretical growth function can be developed to explain the growth as a function of stand age and the level of competition. However, the actual specification and subsequent estimation of the model parameters are based on data consisting of a large cross-section of individual trees from different sites and stand densities making up the population. It is not self-evident that an individual growth function could be developed to model the growth of individual trees over a wide range of stand conditions (Theil 1971, Behr 1981). For a given level of competition, site productivity, and age it would be reasonable to assume that the growth response would differ over a wide range of planting densities. It would be unrealistic to think that one variable such as planting density could be used to estimate the

growth response of an individual tree for various levels of competition over time. In such a case, all sample observations are grouped together and the estimation of the model parameters are performed on the combined sample of cross-sectional and time-series data (Balestra and Nerlov 1966).

Often in pooling data across a wide range of planting densities, it is difficult if not impossible to capture all of the important cross-sectional effects. There may be specific factors associated with each cross-section effect that help explain the observed variation in the dependent variables but are unavoidably excluded from the model (Behr 1981, Balestra and Nerlove 1966).

Inferences on the estimated model parameters and any confidence statements are based on the assumptions pertaining to the properties of the residual vector ε. The specific ignorance due to the exclusion of specific cross-sectional relationships results in overestimating the disturbance term. Each residual ε_{nt} for the nth observation during time period t, may be decomposed into two statistically independent parts:

$$\varepsilon_{nt} = \gamma_{nt} + \mu_n$$

where γ_{nt} is the residual variation due to general ignorance (sampling error), and μ_n is the residual variation due to specific cross-sectional ignorance. It is assumed that γ_{nt} has zero mean and variance σ_γ^2 and covariance matrix $\sigma_\gamma^2 I$. It is further assumed that μ_n is a random variable with mean zero and variance σ_μ^2. Thus the residual ε_{nt} has zero mean and variance $\sigma_\varepsilon^2 = \sigma_\gamma^2 + \sigma_\mu^2$ and covariance $\rho \ \sigma_\varepsilon^2$ of γ_{nt} and $\gamma_{n't'}$ for any pair of observations (nt, n't') where nt≠n't' is equal to σ_μ^2. Hence, $\rho = \sigma_\mu^2/(\sigma_\gamma^2 + \sigma_\mu^2)$. It is further assumed that the cross-sectional

disturbances μ_n are serially independent and that the disturbances are independent from one observation to the next. In other words:

$$E\left(\mu_n, \mu_{n'}\right) = \begin{cases} \sigma_\mu^2 & n = n' \\ 0 & \text{otherwise} \end{cases}$$

$$E\left(\gamma_{nt}, \gamma_{n't'}\right) = \begin{cases} \sigma_\gamma^2 & n = n', \ t = t' \\ 0 & \text{otherwise} \end{cases}$$

$$E\left(\gamma_{nt}, \mu_n\right) = 0 \text{ for all } n, t.$$

The assumption that the cross-sectional disturbances μ_n are independent from one observation to the next needs mentioning. If the sample trees are selected at random and are independent of each other, then this assumption can readily be satisfied. However, if the trees are systematically selected as in this study, this assumption may not be well satisfied and must be taken into consideration when interpreting the results of the analysis.

From the above discussion the variance-covariance matrix can be written as follows:

$$E\left(\varepsilon, \varepsilon'\right) = \Omega = \sigma_\varepsilon^2 \begin{bmatrix} A_1 & 0 & . & . & . & 0 \\ 0 & A_2 & . & . & . & 0 \\ & . & . & . & & \\ . & . & & . & & . \\ . & . & & & . & . \\ 0 & 0 & . & . & . & A_N \end{bmatrix} .$$

where

$$E\left(\varepsilon_n, \varepsilon_n^{\prime}\right) = \sigma_\varepsilon^2 A = \sigma_\varepsilon^2 \begin{bmatrix} 1 & \rho & \rho & \cdot & \cdot & \cdot & \rho \\ \rho & 1 & \rho & \cdot & \cdot & \cdot & \rho \\ & & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & & \cdot & \cdot \\ \rho & \rho & \rho & \cdot & \cdot & \cdot & 1 \end{bmatrix} \quad .$$

is a T x T matrix, and

$$\sigma_\varepsilon^2 = \sigma_\gamma^2 + \sigma_\mu^2$$

$$\rho = \sigma_\mu^2 / \sigma_\varepsilon^2 \ .$$

In a single equation model with a cross-sectional time-series component, OLS estimates of the coefficient vector are inefficient. Consequently, GLS is required to obtain efficient parameter estimates:

$$\hat{\beta} = (X^\prime \Omega^{-1} X)^{-1} X^\prime \Omega^{-1} Y.$$

However, since Ω is usually not known, it must be estimated from the sample data. A technique often employed to solve for the parameter vector β is to search over all values of ρ and choose the corresponding parameter set which maximizes the likelihood function (Behr 1981).

If ε is distributed according to a multivariate normal distribution with zero mean and covariance matrix Ω, then the probability density function $P(\varepsilon)$ may be written as follows:

$$P(\varepsilon) = (2\pi)^{-NT/2} \left| \Omega^{-1} \right|^{1/2} \exp\left(-\frac{1}{2} \varepsilon^\prime \Omega^{-1} \varepsilon \right)$$

where $\varepsilon = y - X\beta$. The logarithmic likelihood function for the parameters ρ, γ and σ_ε^2 may be written as follows:

$$L\left(\rho, \gamma, \sigma_\varepsilon^2\right) = \frac{-NT}{2} \, Ln(2\pi) \ - \frac{1}{2} \, Ln\left| \Omega^{-1} \right| \ - \frac{1}{2} \, \varepsilon^\prime \Omega^{-1} \varepsilon.$$

The maximum of the likelihood function $L(\rho,\gamma,\sigma_\varepsilon^2)$ is obtained when the quadratic form

$$\varepsilon'\Omega^{-1}\varepsilon = [y - X\gamma]\Omega^{-1}[y - X\gamma]$$

is minimized. This occurs when

$$\hat{\gamma} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}y.$$

The value for σ_ε^2 may be found by differentiating $L(\rho,\gamma,\sigma_\varepsilon^2)$ with respect to σ_ε^2 and setting the results to zero.

Behr (1981) presented an alternative maximum likelihood estimator called the concentrated likelihood estimator which is given by

$$-2L(\rho,\gamma,\sigma_\varepsilon^2) = \sum^N T_i Ln(2\Pi) + \sum^N T_i Ln(\sigma_\varepsilon^2(1-\rho)) + \sum^N Ln(\frac{1-\rho + T_i\rho}{1-\rho}) \ .$$

Using the GLS procedure, estimates of σ_ε^2 are obtained for each value of ρ along with the corresponding estimates of γ. The value of ρ that maximizes the concentrated maximum likelihood function is the maximum likelihood estimate of ρ which implies that the corresponding estimates of γ and σ_ε^2 are also maximum likelihood estimates (Maddala 1977, Behr 1981, Balestra and Nerlove 1966, Theil 1971). Since ρ varies over the closed interval, zero to one, $L(\rho,\gamma,\sigma_\varepsilon^2)$ must reach a maximum within the interval or at one of the boundaries. If the maximum likelihood estimator of ρ is zero, this implies that the cross sectional error is equal to zero $\sigma_\mu^2 = 0$ and that the total variability of the model $\sigma_\varepsilon^2 = \sigma_\gamma^2$ is due to general ignorance and not to a specification error in the model.

In the presence of a cross-sectional time-series error component, the normal 2SLS procedure will be inefficient. The suggested procedure

is to apply the ML estimator to each of the four first stage structural equations (Maddala 1977, Behr 1981). For each equation, search over the values of ρ and choose the ρ value and corresponding parameter set that maximizes the concentrated likelihood function. Thus, for each equation a set of modified 2SLS estimates and corresponding ρ values are obtained. The third stage is then applied to the data using the variance-covariance matrix obtained in the second stage.

Model Validation

The efficiency of the OLS and 3SLS estimation procedure was evaluated by four goodness-of-fit statistics (Murphy 1983):

Fit Index,
$$R^2 = 1 - \sum^n \frac{(\hat{Y}_i - Y_i)^2}{(Y_i - \overline{Y})^2}$$

Mean Difference,
$$\overline{D} = \frac{1}{n} \sum^n (\hat{Y}_i - Y_i)$$

Root Mean Square Error,
$$RMSE = \left[\frac{1}{n} \sum^n (\hat{Y}_i - Y_i)^2 \right]^{1/2}$$

Percent Mean Difference,
$$\overline{D\%} = \frac{1}{n} \sum^n (\frac{\hat{Y}_i - Y_i}{Y_i}) \, 100$$

Percent Root Mean Square Error,
$$RMSE\% = \left[\frac{1}{n} \sum^n (\frac{\hat{Y}_i - Y_i}{Y_i})^2 \right]^{1/2} 100$$

where

n = number of observations

Y_i = observed value

$\overline{Y}$ = mean of observed values

$\hat{Y}_i$ = predicted value.

Other checks on the validity of the growth model included

(a) An examination of the model coefficients for wrong signs and associated variance inflation factors (VIF). The VIF is defined as the inverse of partial correlation coefficients associated with the dependent variables in the model,

(b) Comparison of the correlations of errors across equations for the 3SLS procedure, and

(c) The efficiency of the 3SLS procedure relative to the OLS estimation procedure. An F-test was used to examine the equality of variances of the two estimation procedures:

$$F_{ij} = \frac{\sigma^2_{OLS(\hat{\beta})}}{\sigma^2_{3SLS(\hat{\beta})}}$$

where F_{ij} is the F-statistic for the ith equation and jth parameter.

RESULTS AND DISCUSSION

Spatial Analysis

Hopkins' modified coefficient of aggregation was used to depict the spatial pattern of healthy and dead trees in the study area at four years of age. Hopkins' index yields a point estimate of the spatial distribution ranging from 0.2 to 0.87 (fairly uniform to highly aggregated). A value of 0.5 corresponds to a random spatial pattern. A population is said to be distributed at random if the relative location of an individual tree is independent of the location of any other tree within an infinitely large area. An aggregated population is one in which the trees occur in clumps of varying densities and sizes. A uniform spatial distribution is one in which the trees are evenly distributed over a given area on the vertices of a triangular, square or hexagonal lattice (Reich 1980).

Two transition levels, occurring at approximately 0.4 and 0.6, separate the three basic spatial patterns, uniform, random and aggregated. Values near the transition levels are not clearly discriminating. Following is a discussion of the spatial patterns of healthy and dead trees observed in the study.

At four years of age healthy trees were uniformly distributed throughout the study area (Table 2). But significant differences were observed in the spatial patterns of healthy trees among spacings and

replications (Table 3). The significant interaction suggests a lack of consistency of spacing and replication effects. However, the effect was very small. All but one of the spacings had higher coefficients in replication A than in replication B. The significant replication suggests a location effect (Table 2). In general, the spatial patterns of healthy trees were significantly more uniformly distributed at wider spacings. It should be noted that Hopkins' index is independent of tree density and that the index is actually measuring the distribution and size of the vacancies created by mortality. Since the survival rate increased with spacing, the wider spacings retained more of the original planting configuration than the smaller spacings which had a lower survival rate.

The spatial patterns of dead trees were randomly distributed throughout the study area, but varied significantly among spacings and replications (Table 2 and 4). No spacing (x) replication interaction was observed. In five of the 12 plots a transition was observed among spatial patterns for the dead trees. Four of the transitions were from random to uniform while one was from random to aggregate. The transitions between spatial patterns are indicated by a two letter code: U/R, R/U, A/R, and R/A. The sequence of letters is based on the frequency of the observed spatial pattern. For example R/U indicates that at least 50 percent (or five out of ten) of the observed spatial patterns were random while the remaining estimates indicated a uniform spatial pattern.

As mortality in the stand continues over time, the spatial distribution of the healthy and dead trees will change with time. In general, healthy trees will tend to converge from uniform to more of a

Table 2. Spatial analysis for healthy and dead trees at four years of age.

Spacing	Rep	Healthy[1] Trees	Spatial[3] Pattern	Dead[1] Trees	Spatial Pattern
5	A	$.3277^{a,2}$ [2]	U	$.4899^{cd,1}$	R
	B	$.3352^{a,2}$	U	$.4335^{cde,1}$	R
7	A	$.2829^{b,2}$	U	$.5255^{abc,1}$	R
	B	$.2387^{c,2}$	U	$.3966^{de,1}$	R/U
9	A	$.2161^{c,2}$	U	$.5085^{cd,1}$	R
	B	$.1888^{e,2}$	U	$.5202^{ac,1}$	R
11	A	$.2337^{c,2}$	U	$.4503^{cde,1}$	R/U
	B	$.2008^{de,2}$	U	$.4011^{dc,1}$	U/R
13	A	$.1002^{f,2}$	U	$.6247^{a,1}$	R/A
	B	$.0994^{f,2}$	U	$.6240^{ab,1}$	R
15	A	$.2359^{c,2}$	U	$.4442^{cde,1}$	R
	B	$.2266^{c,2}$	U	$.3392^{e,1}$	U/R

[1] Mean value for Hopkins' modified coefficient of aggregation, A^{*}, based on ten independent samples of 150 point-to-tree and 150 tree-to-tree distances.

[2] Means with different letter superscripts indicate significant differences among spacing and replications. Means with different number superscript indicate significant difference between healthy and dead trees ($\alpha = 0.05$).

[3] U = uniform spatial pattern;   R = random spatial distribution

A = aggregated spatial distribution.

Table 3.   Analysis of variance of the spatial distribution of healthy
            trees in the McArthur spacing study at age four years.

Source	df	SS	MS	F
Spacing (S)	5	.541	.108	173.06**
Replication (R)	1	.009	.009	14.29**
S x R	5	.009	.002	2.92*
Error	98	.061	.001	
TOTAL	109	.620		

** Significant at the α = 0.01 level.

* Significant at the α = 0.05 level.

Table 4.   Analysis of variance of the spatial distribution of dead
           trees in the McArthur spacing study at age four years.

Source	df	SS	MS	F
Spacing (S)	5	.614	.123	11.02**
Replication (R)	1	.079	.079	7.07**
S x R	5	.067	.013	1.21
Error	98	1.080	.011	
Total	109	1.840		

** Significant at the α = 0.01 level

random spatial distribution while the dead trees will converge toward a uniform or aggregated spatial pattern (Kent and Dress 1979, Kent and Dress 1980, Reich 1980). The convergence of spatial patterns over time are evident in certain stands infected with fusiform rust and pitch canker.

Effects of Microsite Differences on Individual Tree Growth

Analysis of variance was used to study the effects of spacing on survival, DBH, total height, cubic volume and bole surface area at 2, 6, 8, 10, 12, 14, 16, 19 and 21 years after planting. Results indicate that survival and tree size (DBH, total height, cubic volume, and surface area) are significantly influenced by spacing. No significant differences were observed between replications; however, there was a significant spacing (x) replication interaction. The significant interaction suggests a lack of consistency of spacing and replication effects and suggests that the location of the plot played an important role in the growth of the individual trees in this study. Even though the 12 acre study area appears to be homogeneous, in reality there seems to be an intricate pattern between "good" and "poor" microsites within the study area.

After correcting DBH, survival, tree heights, cubic volumes, and surface area data for spacing and replication effects and plotting the corrected values over stand age and plot location, a definite microsite pattern emerge which help explain the significant spacing X replication interaction. The effects of microsite on survival, DBH, total height, cubic volume, and surface area are depicted in Figures 2 through 6 by

plot location and stand age. Uncorrected plot means are shown in Figures 7 through 10.

In the center of the study area there is a clearing with very little understory vegetation. East and west of the clearing the understory vegetation increases both in density and height. The dominant understory species are wax myrtle (_Myrica cerifera_ L.) which averaged 15.5 feet in height (Table 5).

At four years of age, survival, DBH, and tree heights were somewhat larger in the center of the study area and decreased in size and numbers along the east-west direction. The observed microsite pattern corresponds approximately to the distribution of the understory vegetation of the study area. The microsite effect on tree heights decreases with age and diminishes by the age of 21 years (Figure 3).

Microsite noticeably influenced DBH, cubic volume, surface area and survival over the 21 year period (Figures 2,4,5, and 6). Over time, the microsite pattern changed; diameters, volumes, surface area and survival rates were higher in the east-west portion of the study area and decreased toward the center. No silvicultural practices, such as burning, have been conducted on the study area since establishment in 1963. Most likely the observed microsite pattern is a biological response caused by microtopography and underground drainage variation.

A recent study by Permar and Fisher (1983) indicated that wax myrtle is an important species for fixing nitrogen in southern flatwoods. It has been shown that nitrogen fertilizer can result in substantial increases in volume growth of slash pine on spodosal soil sites in Southeastern U.S. (Fisher and Garbett 1980). These flatwood soils are generally low in available nitrogen and respond to added

nitrogen with increased growth. Stands fertilized with nitrogen have a gain of approximately 4.5 m^3 /ha/year compared to unfertilized ones (Fisher and Garbett 1980). Also, the nitrogen content of both the litter and soil tend to be greater beneath wax myrtle than pines. Average annual accumulation of nitrogen under wax myrtle was 2.4 g N/m^3 /year (Permar and Fisher 1983).

It may be assumed that the understory vegetation became established shortly after or during the planting of the stand. During the first four to six years, understory species (wax myrtle, in particular) competed with the young pine seedlings in the east and west portion of the stand. This competition resulted in a decrease in both diameter and height growth compared to trees located in the center of the study area where there was not competing understory vegetation. When the pine trees were able to compete with the understory vegetation for growing space, nutrients, light and water, they were able to efficiently utilize the additional nitrogen fixed by the wax myrtle. The net result is a higher survival rate and larger than average trees in the east and west portion of the stand compared to trees in the center where little or no additional nitrogen from the understory vegetation was available.

This hypothesis may explain the microsite patterns observed in survival and tree size, but it still needs to be tested with a study designed specifically to analyze the effects of microsite and/or nitrification of slash pine by wax myrtle on the growth of individual trees over time.

Table 5. Average height of major understory species for the McArthur
study area at 21 years of age.

Understory Species	Average Height (FT)	Range
Acer rubrum L.	1.4	11.3-16.0
Ilex glabra (L.) Gray	6.6	4.9- 8.0
Myrica cerifera L.	15.5	5.0-25.0
Persea borbonia L.	10.9	6.8-15.5
Quercus incana Bartr.	15.0	15.0
Quercus laurifolia Michx.	10.2	4.8-18.0
Quercus nigra L.	13.8	6.8-25.5
Quercus virginiana Mill.	6.4	6.4
Sabal palmetto (Walt.) Lodd. ex. Sholt.	5.7	4.3- 8.0
Serenoa repens (B.) Small	4.2	3.5- 4.7
Vaccinium stamineum L.	11.2	11.2
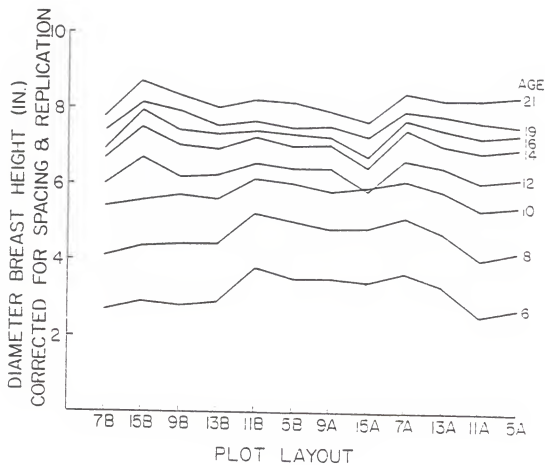
Figure 2. Diameter breast height corrected for spacing and
replication by stand age and plot location.
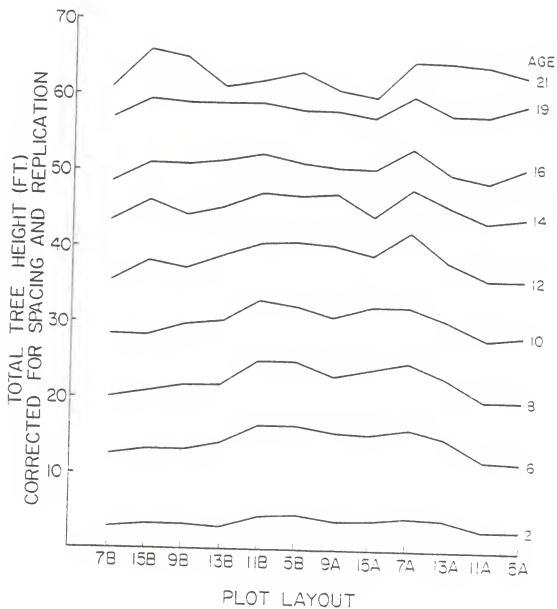
Figure 3. Total tree height corrected for spacing and replication by stand age and plot location.
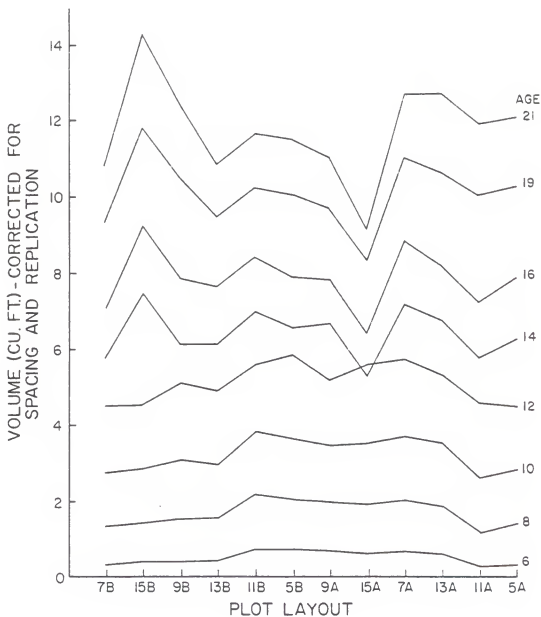
Figure 4. Cubic volume corrected for spacing and replication by stand age and plot location.
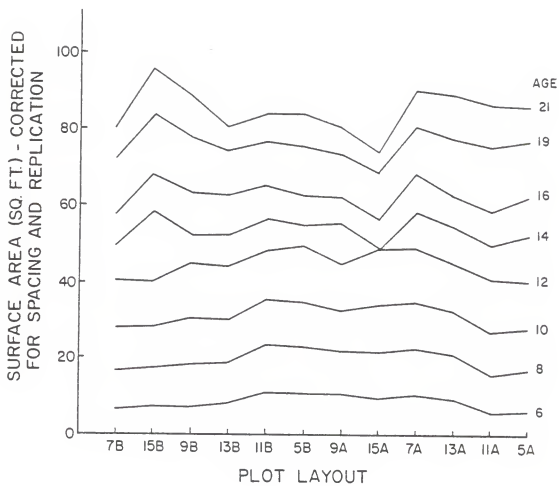
Figure 5. Surface area corrected for spacing and replication by stand age and plot location.
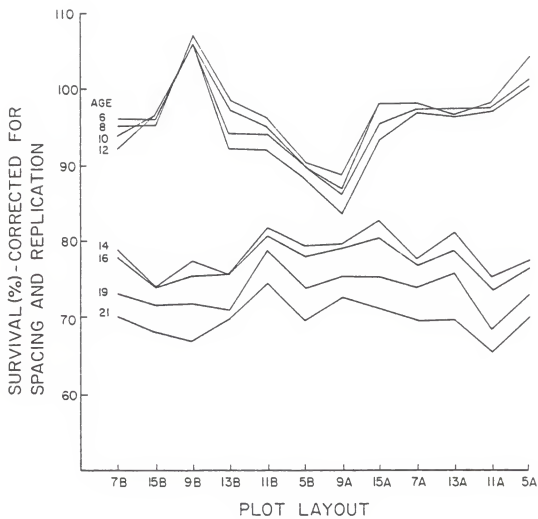
Figure 6. Survival corrected for spacing and replication by stand age and plot location.
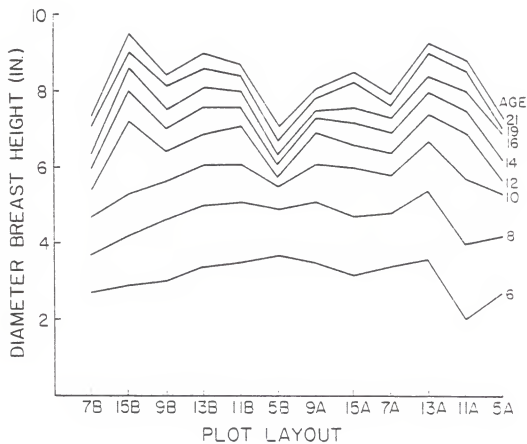
Figure 7. Diameter breast height by stand age and plot location.
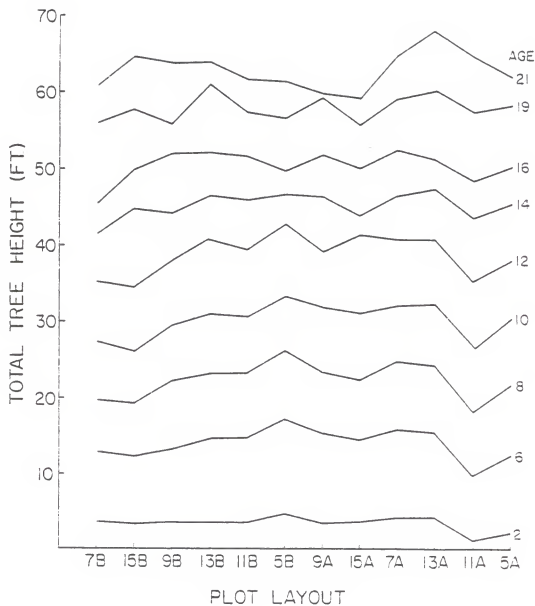
Figure 8. Total tree height by stand age and plot location.
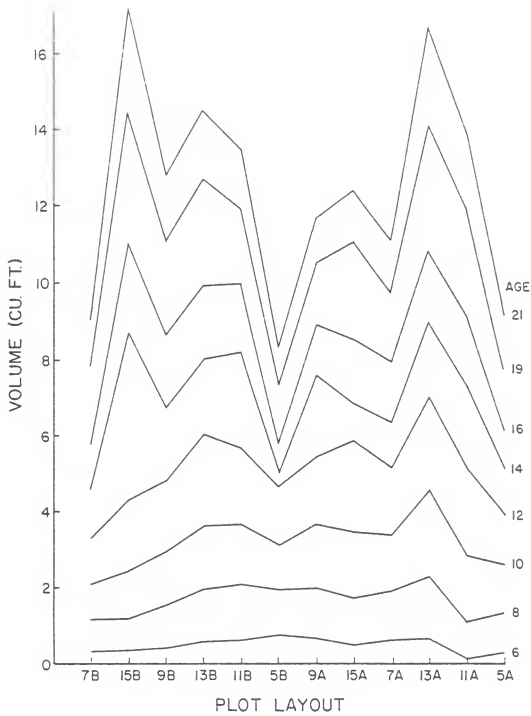
Figure 9. Cubic volume by stand age and plot location.
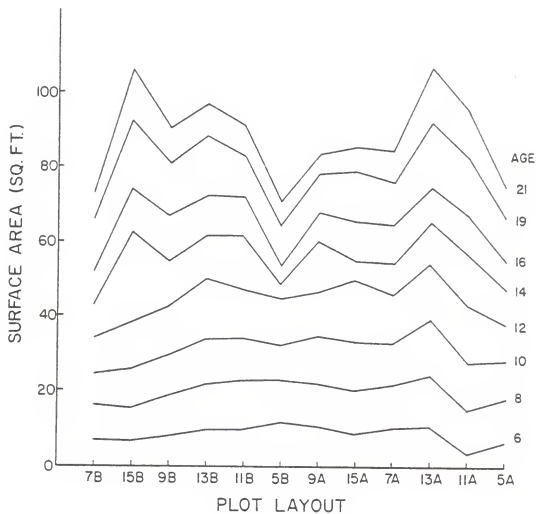
Figure 10. Surface area by stand age and plot location.

Effects of Spacing on Individual Tree Growth

Regression analysis indicates that spacing had no significant effect on height growth after correcting for microsite and replications (Figure 12). However, spacing significantly affected survival rate and size of individual trees, excluding height (Figures 11 and 15). The effects of spacing on survival, DBH, cubic volume and surface area increased with age.

Several studies (Bennett 1974, Shepard 1974) have reported both positive and negative correlations between height growth and spacing. These discrepancies may be in part due to the exclusion of microsite effects from the analysis of the data. In this study conventional analysis of variance indicated that spacing had a significant effect on tree heights. After correcting for microsite and replication effects, regression analysis indicated that spacing had no significant effect on tree heights.

Volume and Surface Area

Regression analysis was used to develop volume and surface area equations to predict cubic foot volumes and bole surface area in square feet for the sample trees used in the study. Regression equations were developed for both inside and outside bark. An exponential equation was used to estimate total cubic volume and surface area as a function of diameter at breast height outside bark (DBH) and total tree height (THT). The form of the equations was based on the residuals and resulting coefficient of multiple determination. The final form of the
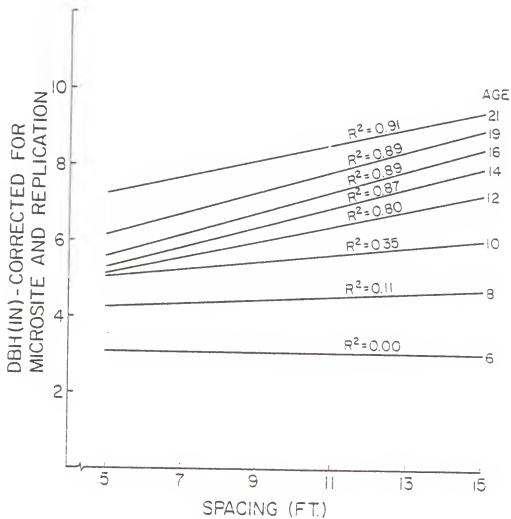
Figure 11. Effects of spacing on diameter breast height corrected for microsite and replication by stand age.
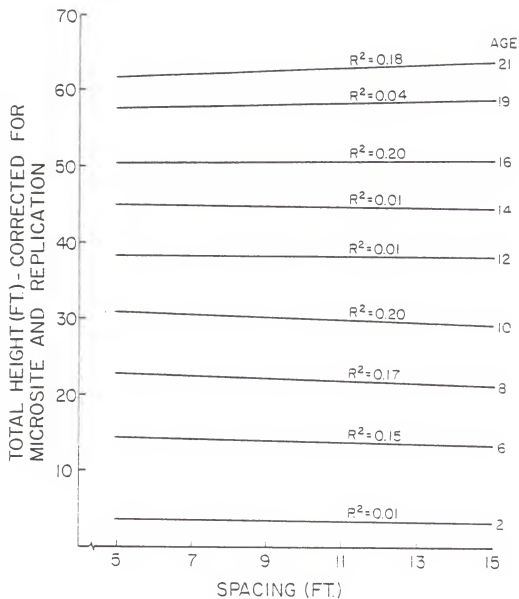
Figure 12. Effects of spacing on total tree height corrected
for microsite and replication by stand age.
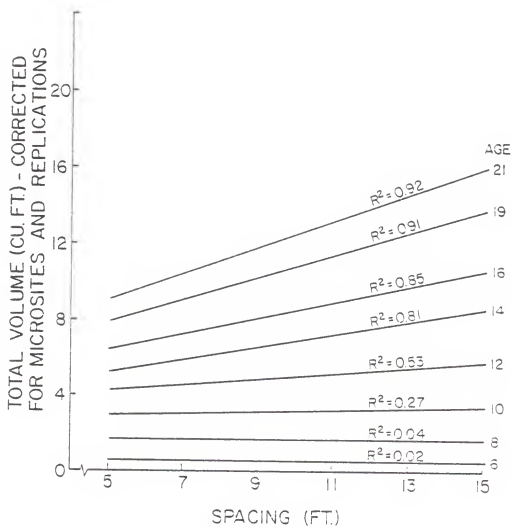
Figure 13. Effects of spacing on cubic volume corrected for microsite and replication by stand age.
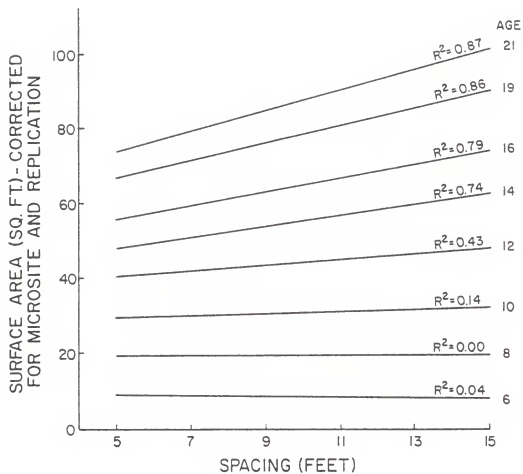
Figure 14. Effects of spacing on surface area corrected for microsite and replication by stand age.
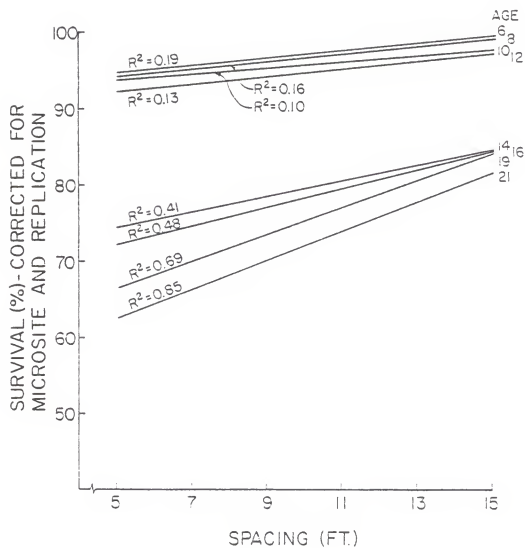
Figure 15. Effects of spacing on survival corrected for microsite and replication by stand age.

equation used to estimate cubic volume and surface area is

$$Y = \alpha DBH^{\beta} THT^{\gamma}$$

where α, β and γ are regression coefficients estimated from the sample
dendrometry data using log transformation.

Volume

$$VOB = \exp[-5.3036+2.1963Ln(DBH)+0.7492Ln(THT)]$$

$$R^2 = 0.970 \qquad s_{Ln(VOB)} = 0.098$$

Relative variation: 0.91 to 1.10

$$VIB = \exp[-5.9612+2.3004Ln(DBH)+0.7928Ln(THT)]$$

$$R^2 = 0.964 \qquad s_{Ln(VIB)} = 0.114$$

Relative variation: 0.89 to 1.12

Surface Area

$$SOB = \exp[-1.4180+1.1376Ln(DBH)+0.8358Ln(THT)]$$

$$R^2 = 0.969 \qquad s_{Ln(SOB)} = 0.060$$

Relative variation: 0.94 to 1.06

$$SIB = \exp[-1.7201+1.1859Ln(DBH)+0.8559Ln(THT)]$$

$$R^2 = 0.966 \qquad s_{Ln(SIB)} = 0.066$$

Relative variation: 0.94 to 1.07

where

VOB = cubic foot volume outside bark

VIB = cubic foot volume inside bark

SOB = bole surface area outside bark

SIB = bole surface area inside bark

exp = exponential

Ln = natural logarithm.

Cubic volume and bole surface area outside bark by DBH and total height are given in Tables 6 and 7.

Tree Form Equations

Substituting the cubic volume and surface area equations into Grosenbaugh's (1966) four ratios involving solids of revolution yields the following relationship between tree form, diameter at breast height and total height for the sample trees used in this study:

Outside Bark

$$Q_o = 0.9549 (DBH^{2.196} \ THT^{-0.250}) 1/2$$

$$A_o = 0.9251 \ DBH^{1.138} \ THT^{-0.164}$$

$$Q_o/A_o = 0.9857 \ DBH^{1.059} \ THT^{-0.087}$$

$$F_o = 0.927 \ DBH^{0.079} \ THT^{-0.076}$$

Table 6. Total cubic foot volume outside bark for slash pine, of the McArthur study area.

DBH Class (IN)	Total Height (ft)														
	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
4	1.16	1.34	1.50	1.66	1.81	1.96	2.10	2.24	2.38	2.52					
5	1.90	2.18	2.45	2.70	2.96	3.20	3.43	3.66	3.89	4.11	4.33				
6		3.25	3.65	4.04	4.41	4.77	5.12	5.47	5.81	6.14	6.46	6.78	7.10		
7			5.12	5.66	6.18	6.69	7.19	7.67	8.15	8.61	9.07	9.52	9.96		
8				7.59	8.29	8.97	9.64	10.29	10.92	11.55	12.16	12.56	13.35		
9				9.83	10.74	11.62	12.48	13.32	14.15	14.96	15.75	16.53	17.30	18.05	
10					13.54	14.65	15.73	16.79	17.83	18.85	19.85	20.83	21.80	22.75	
11						18.06	19.40	20.70	21.98	23.24	24.47	25.68	26.88	28.05	29.21
12							23.48	25.06	26.61	28.13	29.62	31.09	32.54	33.96	35.36
13							28.00	29.88	31.73	33.54	35.32	37.07	38.79	40.49	42.16
14								35.16	37.34	39.47	41.56	43.62	45.65	47.64	49.61
15								40.92	43.44	45.92	48.36	50.76	53.11	55.44	57.73

VOL. = exp [−5.30357173 + 2.19634063 Ln (DBH) + 0.74915347 Ln (HT)]

Standard Error of Estimate = 0.098
Blocks indicate the extent of basic data
Number of trees 146
Dendrometry data gathered with Barr and Stroud Optical Dendrometer
R**2 = 0.970.

Table 7. Total bole surface area outside bark for slash pine of the McArthur study area.

DBH Class (IN)	Total Height (ft)														
	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
4	17.27	20.12	22.89	25.59	28.24	30.84	33.40	35.91	38.40	40.85					
5	22.27	25.94	29.50	32.99	36.40	39.75	43.05	46.29	49.50	52.66	55.78	58.88			
6		31.91	36.30	40.59	44.79	48.91	52.97	56.96	60.90	64.80	68.64	72.45	76.21		
7			43.26	48.37	53.37	58.29	63.13	67.88	72.58	77.22	81.80	86.33	90.82		
8				56.30	62.13	67.85	73.47	79.02	84.48	89.88	95.22	100.50	105.72		
9				64.38	71.04	77.58	84.01	90.35	96.60	102.77	108.87	114.90	120.88	126.79	
10					80.08	87.46	94.71	101.85	108.90	115.86	122.73	129.54	136.27	142.94	
11						97.47	105.55	113.52	121.37	129.12	136.79	144.37	151.88	159.37	166.67
12							116.54	125.33	134.00	142.56	151.02	159.39	167.68	175.88	184.01
13							127.65	137.27	146.77	156.15	165.42	174.59	183.66	192.65	201.55
14								149.35	159.68	169.89	179.97	189.94	199.82	209.60	219.28
15								161.54	172.72	183.76	194.66	205.45	216.13	226.71	237.19

$$SUR = exp [-1.418 + 1.1376 Ln (DBH) + 0.8358 Ln (THT)]$$

Standard Error of Estimate = 0.060
Blocks indicate the extent of basic data
Number of trees 146
Dendrometry data gathered with Barr and Stroud Optical Dendrometer
$R**2 = 0.969$.

Inside Bark

$$Q_I = 0.6873(DBH^{2.300} \ THT^{-0.207}) \frac{1}{2}$$

$$A_I = 0.6839 \ DBH^{1.186} \ THT^{-0.144}$$

$$Q_I/A_I = 0.6908 \ DBH^{1.114} \ THT^{-0.063}$$

$$F_I = 0.9880 \ DBH^{0.071} \ THT^{-0.081}.$$

Table 8 compares average tree form, inside and outside bark, by
diameter and total height.

Parameter Estimation

The growth model for diameter, total height, cubic volume and
surface area was fitted to a sample of 2600 cross-sectional and time-
series observations using a 3SLS procedure. Although the third-stage
estimates are the parameters of interest, the results of the first and
second stages will also be discussed.

First Stage Estimates

In the first stage, estimates are obtained for the endogenous
variables (diameter and total height) which appear as explanatory ones
in the volume and surface area equations. This is accomplished by
regressing all predetermined variables on diameter and height using
OLS. The estimated equations represent the unrestricted reduced form
parameter estimates for diameter and total height. The predicted values
for diameter and total height are uncorrelated with the disturbances
associated with the cubic volume and surface area equations.

Table 8. Comparison of tree form inside (F_I) and outside (F_o) bark by diameter and total height.

	Total Height						
	Inside Bark (F_I)				Outside Bark (F_o)		
DBH	50	70	90		50	70	90
5	81	79	77		79	77	75
6	82	80	78		80	78	76
7	83	81	79		81	79	77
8	84	81	80		82	80	78
9	84	82	80		82	80	79
10	85	83	81		83	81	79
11	86	83	82		84	82	80
12	86	84	82		84	82	81
13	87	84	83		85	83	81
14	87	85	83		85	83	82
15	87	85	83		86	84	82

Second Stage Estimates

In the second stage, maximum likelihood estimates of the structural parameter vector and variance-covariance matrix are obtained based on the assumptions regarding the cross-sectional and time-series characteristics of the disturbances. The endogenous explanatory variables are replaced by the predicted values for diameter and total height, which are obtained from the first stage. The GLS procedure was used to estimate the structural parameters of the growth model for the second stage. The variance-covariance matrix was estimated iteratively.

Because of the large data set (N1=2600) (requiring excessive time and space to invert a 2600 square matrix) the total data set (N1+N2=5199) was divided into 52 groups with approximately 100 observations per group. A random sample of 15 groups was selected to obtain a maximum likelihood estimate of ρ. Searching over all values of ρ within the interval 0 to 1 in increments of 0.1, a unique set of structural parameters and model variances was obtained for each equation and sample group. The ML estimate of ρ, and the corresponding model variance and concentrated likelihood function are given in Tables 9 through 12.

The parameter ρ is the ratio of the cross-sectional and time-series error component to the total model variance. All ML estimates of ρ for the four equations and 15 samples were zero. A ρ value of zero implies that the model variance arises from random sampling error with no cross-sectional or time-series error component. Table 13 shows the absolute value of the concentrated likelihood function for the four equations over the range of ρ. Table 14 depicts the relationship between the error sum of squares and the ML estimate of ρ. The error sum of squares

increases exponentially as ρ approaches one, while the concentrated maximum likelihood function decreases in a somewhat linear fashion. The estimated structural parameters and model variance are very stable near the region of the ML value of ρ. The changes in the estimated parameters for the diameter equation are given in Table 15 for selected values of ρ.

Although the ML estimator has many desirable properties, there have been questions concerning its use in the variance-components models (Behr 1981). Monte Carlo simulations have shown that the estimated ρ values were zero when the true value was actually larger. It was argued that the serial nature of the exogenous variables interacted strongly with the autoregressive nature of the problem under study, causing problems in estimating ρ. This was not considered a problem in this study.

Third Stage Estimates

In the third stage an asymptotically efficient estimate of the growth model is obtained by regressing the four equations together as a system. Since the ML estimate of ρ was zero, the standard 3SLS procedure was used to obtain the third stage estimates of the parameters. Since the disturbances from the four equations are contemporaneously correlated with variance-covariance matrix Ω, and since Ω is not known, it is estimated from the second stage using data set N1. In the second stage each equation is transformed in accordance with the GLS procedure when Ω is unknown.

Table 9. Cross-sectional time-series adjustments for diameter
at breast height

RUN	OBS	T[1]	ρ	CONLK[2]	RSQ	VAR	ESS[3]
1	100	9	0	1.329	0.974	0.013	1.196
2	100	9	0	1.878	0.964	0.019	1.690
3	100	9	0	2.122	0.965	0.021	1.909
4	100	9	0	2.477	0.956	0.024	2.229
5	100	9	0	1.981	0.965	0.020	1.783
6	100	9	0	2.902	2.943	0.029	2.612
7	100	9	0	4.073	0.920	0.041	3.666
8	100	9	0	2.859	0.934	0.029	2.573
9	100	9	0	3.518	0.935	0.035	3.167
10	100	9	0	1.405	0.971	0.014	1.265
11	100	9	0	4.293	0.935	0.043	3.863
12	100	9	0	1.413	0.974	0.014	1.272
13	100	9	0	2.800	0.937	0.028	2.520
14	100	9	0	1.860	0.958	0.019	1.674
15	100	9	0	4.191	0.909	0.042	3.772

[1] Number of time periods

[2] Concentrated likelihood value

[3] Error sum of squares

Table 10. Cross-sectional time-series adjustments for total tree height.

RUN	OBS	T[1]	ρ	CONLK[2]	RSQ	VAR	ESS[3]
1	100	9	0	1.031	0.985	0.010	0.928
2	100	9	0	0.845	0.988	0.008	0.761
3	100	9	0	1.353	0.981	0.014	1.218
4	100	9	0	1.373	0.982	0.014	1.235
5	100	9	0	1.096	0.985	0.011	0.986
6	100	9	0	3.412	0.947	0.034	3.071
7	100	9	0	3.151	0.953	0.032	2.836
8	100	9	0	0.798	0.988	0.008	0.718
9	100	9	0	3.208	0.955	0.032	2.887
10	100	9	0	0.833	0.987	0.008	0.750
11	100	9	0	3.342	0.956	0.033	3.008
12	100	9	0	1.230	0.983	0.012	1.102
13	100	9	0	0.968	0.985	0.010	0.871
14	100	9	0	0.914	0.985	0.009	0.823
15	100	9	0	4.178	0.930	0.042	3.760

[1] Number of time periods

[2] Concentrated likelihood estimate

[3] Error sum of squares

Table 11. Cross-sectional time-series adjustments for cubic foot volume.

RUN	OBS	T[1]	ρ	CONLK[2]	RSQ	VAR	ESS[3]
1	100	9	0	3.494	0.986	0.035	3.144
2	100	9	0	4.626	0.983	0.046	4.164
3	100	9	0	9.248	0.968	0.092	8.323
4	100	9	0	6.907	0.974	0.069	6.216
5	100	9	0	8.283	0.970	0.083	7.455
6	100	9	0	13.562	0.945	0.136	12.205
7	100	9	0	20.176	0.921	0.202	18.158
8	100	9	0	5.391	0.974	0.054	4.852
9	100	9	0	17.360	0.933	0.174	15.624
10	100	9	0	4.718	0.980	0.047	4.246
11	100	9	0	21.632	0.934	0.216	19.469
12	100	9	0	6.026	0.978	0.060	5.423
13	100	9	0	5.347	0.976	0.053	4.812
14	100	9	0	5.327	0.974	0.053	4.795
15	100	9	0	17.511	0.921	0.175	15.760

[1] Number of time periods

[2] Concentrated likelihood estimate

[3] Error sum of squares

Table 12. Cross-Sectional time-series adjustments for bole surface
area.

RUN	OBS	T[1]	ρ	CONLK[2]	RSQ	VAR	ESS[3]
1	100	9	0	1.459	0.989	0.015	1.313
2	100	9	0	1.829	0.988	0.017	1.556
3	100	9	0	3.747	0.975	0.037	3.372
4	100	9	0	3.031	0.979	0.030	2.728
5	100	9	0	3.302	0.978	0.033	2.972
6	100	9	0	6.672	0.949	0.067	6.005
7	100	9	0	9.098	0.933	0.091	8.188
8	100	9	0	2.604	0.978	0.026	2.344
9	100	9	0	8.880	0.937	0.089	7.992
10	100	9	0	1.825	0.985	0.018	1.643
11	100	9	0	9.181	0.946	0.092	8.263
12	100	9	0	2.346	0.984	0.023	2.111
13	100	9	0	2.134	0.982	0.021	1.920
14	100	9	0	2.328	0.980	0.023	2.096
15	100	9	0	8.659	0.927	0.087	7.793

[1] Number of time periods

[2] Concentrated likelihood estimate

[3] Error sum of squares

Table 13. Concentrated maximum likelihood values for DBH (D), total
tree height (H), cubic volume (V), and surface area (S).

ρ value	Equation			
	D	H	V	S
0	1.9811	1.0955	8.2831	3.3018
.1	8.1684	7.2392	14.3705	9.4326
.2	11.6820	10.7407	17.8556	12.9275
.3	14.3836	13.4356	20.5429	15.6189
.4	16.7478	15.7965	22.8980	17.9766
.5	19.0006	18.0474	25.1439	20.2247
.6	21.3093	20.3552	27.4470	22.5300
.7	23.8691	22.9149	30.0016	25.0873
.8	27.0328	26.0792	33.1602	28.2491
.9	31.8420	30.8897	37.9641	33.0570

Table 14. Error sum of squares for DBH (D), total tree height (H), cubic volume (V), and surface area (S).

	Equation			
ρ value	D	H	V	S
0	1.7830	0.9860	7.4548	2.9717
.1	1.9641	1.0349	8.1662	3.2283
.2	2.2014	1.1418	9.1467	3.6026
.3	2.5099	1.2911	10.4290	4.0982
.4	2.9228	1.4958	12.1481	4.7659
.5	3.5017	1.7859	14.5597	5.7051
.6	4.3703	2.2236	18.1800	7.1170
.7	5.8178	2.9552	24.2150	9.4724
.8	8.7116	4.4204	36.2850	14.1850
.9	17.3893	8.8187	72.4880	28.3240

Table 15. Comparison of the estimated coefficients for the DBH
equation at three levels of ρ.

COEFFICIENT	ρ value		
	.0	.5	.9
INTERCEPT	1.7216	3.7653	4.4884
DBH LAGGED	0.8446	0.8338	0.8278
THT LAGGED	0.3196	0.3259	0.3291
PAR	−0.3201	−0.3032	−0.2783
PAR**2	0.0950	0.0885	0.0815
ALAG/AGE	−2.3077	−5.1239	−6.3334
AGE	0.3117	0.0581	0.0791
(1/AGE)**2	−58.2769	−70.0511	−74.1267
COMP.	−0.0021	−0.0042	−0.0096
DBH MICROSITE	0.3624	0.0342	0.0339

The third stage parameter estimates along with the second stage and the unrestricted OLS estimates of the structural equations are given in Tables 16 through 19. The model has a reasonable level of explanatory power with a weighted R^2 value for the system of approximately 93 percent.

Reduced Form Equations

The reduced form of the system of equations describes the endogenous variables in terms of the current exogenous variables and lagged endogenous and exogenous ones. The coefficients associated with the reduced form equations are given in Table 20 for the individual tree growth model. The reduced form coefficients for the diameter and height equations are identical to the coefficients in the structural equations. This is due to the fact that the two equations do not contain endogenous variables as explanatory variables.

Final Form Equations

The final form describes the endogenous variables in terms of the current and lagged exogenous variables. If we let y_t be the vector of endogenous variables in period t, and X_t the vector of exogenous variables for the same time period, the reduced form can be expressed in the following form:

$$y_t = d_o + D_1 Y_{t-1} + D_2 X_t + D_3 X_{t-1} + \varepsilon_t^*$$

where d_o is a L x 1 element vector of constant terms in the reduced form

Table 16. Estimates of the model parameters and standard errors
for the DBH equation by OLS, 2SLS, and 3SLS.

| PARAMETER | Estimate (S.E. x 10^2) | | |
	OLS	2SLS	3SLS
R**2	0.937	0.934	0.933
N	2599	2600	2600
INTERCEPT	6.204 (57.82)	5.215 (57.89)	8.864 (49.60)
DBH	----	----	----
TOTAL HEIGHT	----	----	----
DBH LAGGED	0.903 (1.40)	0.829 (1.43)	0.862 (1.39)
THT LAGGED	0.277 (1.93)	0.328 (1.91)	0.289 (1.86)
PAR	-0.232 (2.86)	-0.307 (2.84)	-0.296 (2.82)
PAR**2	0.057 (1.16)	0.083 (1.17)	0.079 (1.16)
ALAG/AGE	-8.047 (71.19)	-6.822 (71.25)	-11.346 (60.93)
AGE	0.071 (0.45)	0.066 (0.45)	0.091 (.40)
AGE**2	----	----	----
(1/AGE)**2	-89.584 (404.49)	-79.830 (403.37)	-104.668 (339.38)
(1-ALAG/AGE)*PAR	----	----	----
COMP.	-0.020 (0.75)	-0.020 (0.78)	-0.029 (.058)
COMP./SPACING	----	----	----
DBH MICROSITE	0.035 (1.26)	0.013 (1.28)	0.029 (0.70)

Table 17. Estimates of the model parameters and standard errors
for the total height equation of OLS, 2SLS, and 3SLS.

| PARAMETER | Estimate (S.E. x 10^2) | | |
	OLS	2SLS	3SLS
R**2	0.959	0.964	0.964
N	2599	2600	2600
INTERCEPT	-1.305 (16.99)	-1.387 (15.47)	-1.430 (14.20)
DBH	----	----	----
TOTAL HEIGHT	----	----	----
DBH LAGGED	0.131 (1.26)	0.134 (1.20)	0.157 (1.13)
THT LAGGED	0.744 (1.75)	0.725 (1.58)	0.701 (1.54)
PAR	0.079 (3.29)	0.057 (3.03)	0.114 (2.85)
PAR**2	-0.017 (1.14)	-0.013 (1.05)	-0.019 (1.04)
ALAG/AGE	1.775 (32.75)	2.008 (30.20)	2.199 (25.85)
AGE	-0.044 (1.62)	-0.052 (1.49)	-0.067 (1.25)
AGE**2	0.001 (0.04)	0.001 (0.04)	0.001 (0.03)
(1/AGE)**2	----	----	----
(1-ALAG/AGE)*PAR	-0.140 (7.70)	-0.070 (7.10)	-0.244 (5.42)
COMP.	----	----	----
COMP./SPACING	0.123 (6.25)	0.104 (5.50)	0.154 (4.74)
DBH MICROSITE	----	----	----

Table 18. Estimates of the model parameters and standard errors
for the cubic volume equation by OLS, 2SLS and 3SLS.

PARAMETER	Estimate (S.E. x 10^2)		
	OLS	2SLS	3SLS
R**2	0.996	0.993	0.993
N	2599	2600	2600
INTERCEPT	-0.790 (30.67)	-2.924 (38.07)	1.428 (16.46)
DBH	1.338 (00.001)	1.063 (1.57)	1.088 (1.49)
TOTAL HEIGHT	0.744 (1.21)	1.135 (2.37)	1.096 (2.28)
DBH LAGGED	----	----	----
THT LAGGED	----	----	----
PAR	0.119 (2.09)	-0.185 (2.72)	-0.213 (2.66)
PAR**2	-0.029 (0.72)	0.053 ().90)	0.063 (0.90)
AGE	0.023 (0.25)	0.032 (0.31)	0.068 (0.21)
ALAG/AGE	2.370 (37.96)	4.636 (46.68)	-0.756 (19.99)
AGE**2	----	----	----
1/AGE**2	28.048 (197.41)	36.817 (237.65)	9.074 (89.84)
(1-ALAG/AGE)*PAR	-0.138 (4.86)	0.189 (5.91)	0.136 (5.55)
COMP.	----	----	----
COMP./SP	0.308 (4.01)	0.306 (4.75)	0.146 (4.30)
DBH MICROSITE	----	----	----

Table 19. Estimates of the model parameters and standard errors
for the surface area equation by OLS, 2SLS and 3SLS.

| PARAMETER | Estimate (S.E. x 10^2) | | |
	OLS	2SLS	3SLS
R**2	0.998	0.997	0.997
N	2599	2600	2600
INTERCEPT	2.052	1.728	1.584
	(3.27)	(4.41)	(4.19)
DBH	0.780	0.649	0.661
	(0.42)	(0.75)	(0.72)
TOTAL HEIGHT	0.734	0.924	0.903
	(0.58)	(1.14)	(1.09)
DBH LAGGED	----	----	----
THT LAGGED	----	----	----
PAR	0.040	−0.106	−0.116
	(0.996)	(1.31)	(1.28)
PAR**2	−0.010	0.029	0.033
	(0.34)	(0.43)	(0.43)
AGE	0.076	0.107	0.048
	(0.41)	(0.51)	(0.21)
ALAG/AGE	−2.162	−2.136	−1.373
	(6.88)	(8.28)	(5.45)
AGE**2	−0.001	−0.002	−0.0002
	(0.01)	(0.01)	(0.01)
1/AGE**2	----	----	----
(1−ALAG/AGE)PAR	−0.033	0.117	0.096
	(2.33)	(2.84)	(2.70)
COMP.	----	----	----
COMP./SP	0.113	0.102	0.043
	(1.85)	(2.22)	(2.08)
DBH MICROSITE	----	----	----

Table 20. Reduced form coefficients for the DBH (D), total height
(H), cubic volume (V), and surface area (S) equations.

VARIABLE	EQUATION			
	D	H	V	S
INTERCEPT	8.864	−1.430	9.519	6.150
DBH LAGGED	0.862	0.157	1.111	0.711
THT LAGGED	0.289	0.701	1.083	0.824
PAR	−0.296	0.114	−0.411	−0.209
PAR**2	0.079	−0.019	0.127	0.067
ALAG/AGE	−11.346	2.199	−10.708	−6.884
AGE	0.091	−0.067	0.094	0.047
AGE**2	0.0	0.001	0.002	0.001
1/AGE**2	−104.668	0.0	−104.957	−69.167
(1−ALAG/AGE)*PAR	0.0	−0.244	−0.131	−0.124
COMP.	−0.029	0.0	−0.031	−0.019
COMP./SPACING	0.0	0.154	0.315	0.182
DBH MICROSITE	0.029	0.0	0.031	0.190

while D_1, D_2, and D_3 are matrices of multiplicative reduced form coefficients (D_1 is a L x L square matrix and D_2 and D_3 are both of the same order, L x K) and ε_t^* is the L element vector of reduced form disturbances in time period t. The final form of the system is obtained by eliminating Y_{t-1} from the right hand side of the equations by substituting y_t lagged one period. Upon substitution, the limit of y_t approaches:

$$y_t = (I-D_1)^{-1}d_o + D_2X_t + \sum_{t=1}^{\infty} D_1^{t-1}(D_1D_2 + D_3)X_{t-1} + \sum_{t=0}^{\infty} D_1^t \varepsilon_{t-1}^*$$

The sequence $\sum_{t=o}^{\infty} D_1^t \varepsilon_{t-1}^*$ converges to zero while the sequence $\sum_{t=1}^{\infty} D_1^{t-1}(D_1D_2 + D_3)$ converges to $(I-D_1)^{-1}(D_2 + D_3)$. The latter sequence is referred to as the "total multipliers" of the system. The elements of D_2 describe the immediate impact of a unit change in the exogenous variables and are known as "impact multipliers". The coefficients associated with the sequence $\sum_{t=1}^{\infty} D_1^{t-1}(D_1D_2 + D_3)$ describe the effects during a latter period and are referred to as "interim multipliers". An approximation of the limiting distribution of the final form may be expressed as follows:

$$y_t = (I - D_1)^{-1}d_o + D_2X_t + (I - D_1)^{-1}(D_1D_2 + D_3)X_{t-1}.$$

The final form coefficients of the system are given in Table 21. Figures 16 through 18 depict the relative change in the interim multipliers over time.

Table 21. Final form coefficients for the DBH (D), total height
(H), cubic volume (V), and surface area (S) equations.

VARIABLES	EQUATION			
	D	H	V	S
INTERCEPT	−562.695	−300.102	−940.395	−641.36
PAR	13.999	7.728	23.505	16.116
PAR**2	−4.525	−2.439	−7.540	−5.162
ALAG/AGE	−11.346	2.199	−10.708	−6.884
AGE	0.091	−0.067	0.094	0.047
AGE**2	0.0	0.001	0.002	0.001
A/AGE**2	−104.668	0.0	−104.957	−69.167
(1−ALAG/AGE)*PAR	0.0	−0.244	−0.131	−0.124
COMP.	2.152	1.130	3.582	2.443
COMP./SPACING	−11.178	−5.353	−17.896	−12.179
DBH MICROSITE	0.029	0.0	0.031	0.190
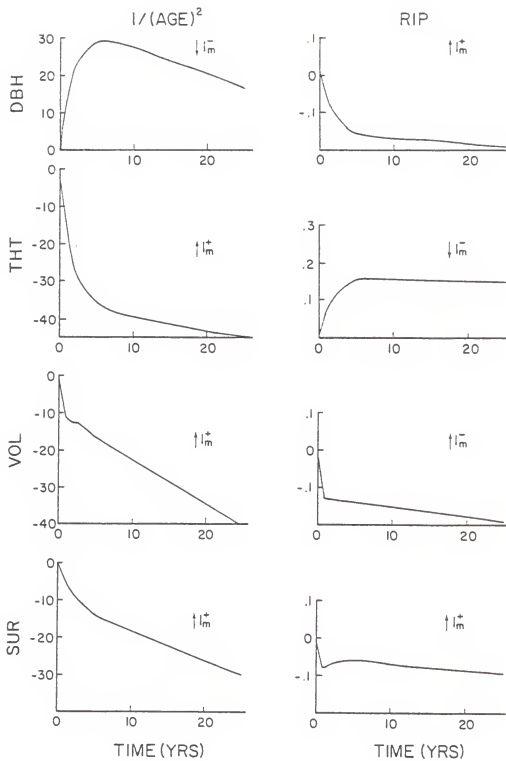
Figure 16. Relative change in the interim multipliers associated with the exogenous variables 1/Age**2 and (1-Alag/Age)* PAR.
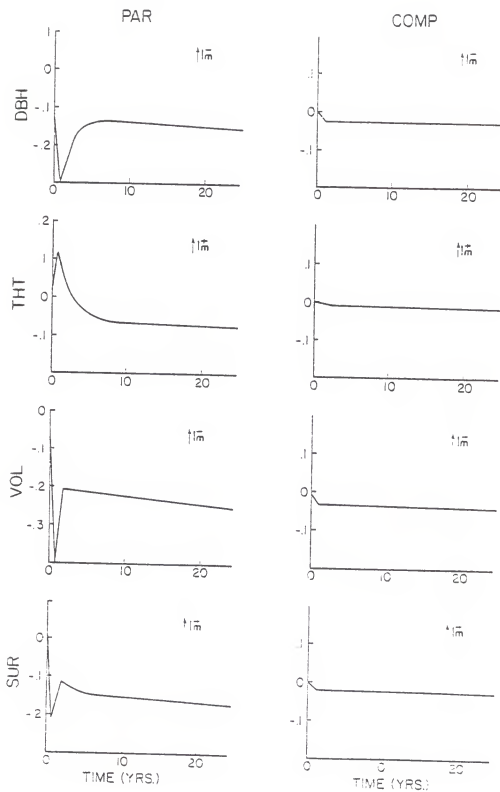
Figure 17. Relative change in the interim multipliers associated with the exogenous variables PAR and Comp.
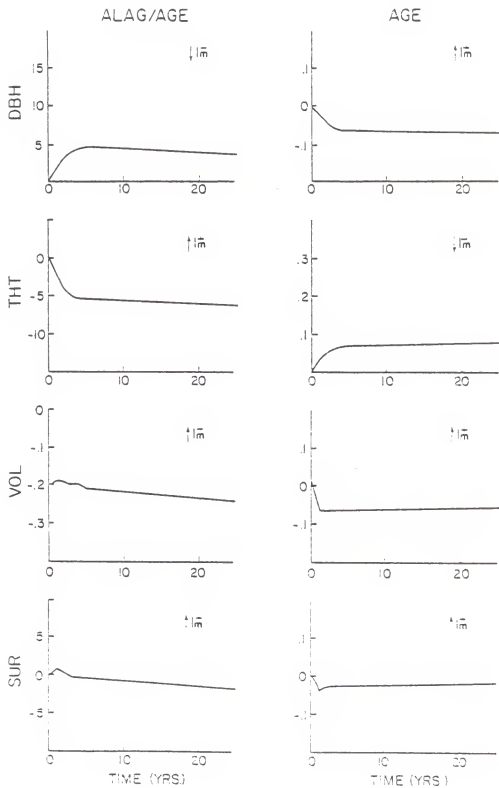
Figure 18. Relative change in the interim multipliers associated
with the exogenous variables Alag/Age and Age.

Model Validation

Regression analysis has widely been used in data analysis and the development of empirical models. Once an adequate model is found, one proceeds to use the model for prediction purposes, process control, or investigation of the mechanisms involved in the developmental or growth processes being studied. For whatever purpose a model is used, some checks on its validity should be made (Brand and Holdaway 1983, Luppold 1982, Reynolds et al. 1981, Snee 1977). A few useful procedures to check the validity of a regression model include:

(a) Comparison of the signs of the estimated coefficients with the theoretical expectations,

(b) Comparison of the results with theoretical models and simulated data, and

(c) Use of an independent data set to obtain a measure of the prediction accuracy of the model.

Evaluation of the Coefficients

An examination of the model coefficients and associated variance inflation factors (VIF) provides a helpful insight into the validity of a model. Coefficients with wrong signs are indications of an inappropriate or poorly estimated model and/or lack of understanding of the system being modeled. The VIF's, the diagonal elements of the inverse of the correlation matrix, measure the collective effect of the correlations among the independent variables on the variance of the estimated coefficients. In general, a VIF larger than ten is an indication that the associated coefficient is likely to be poorly

estimated because of correlations among independent variables. Variables that are highly correlated, generally produce coefficients that are too large and often have wrong signs.

Upon examination of the estimated coefficients for the 3SLS model only two coefficients in the equation for diameters (D) had a wrong sign. The sign of the coefficients indicates that competition has a positive effect on diameters while an increase in microsite had a negative impact on diameters. Competition had a negative effect on height, volume and surface area. The relative spacing of a given tree had a positive effect on diameters, cubic volume and surface areas and a negative impact on heights. Both diameters and heights lagged one period (i.e. two years) had positive effects on diameters and heights. Diameter and total tree height had a positive correlation with cubic volumes and surface areas.

Correlation coefficients among regression coefficients for the four equations are given in Tables 22 through 25; their respective VIF factors are presented in Table 26. The largest VIF factors were associated with the intercept and exogenous variables that included age while the smallest VIF factors were associated with competition and microsite. All but six of the VIF factors were larger than ten, which indicates the coefficients may be poorly estimated because of the existing correlation among independent variables.

The OLS and 3SLS estimates were consistent with respect to the signs of the coefficients for the diameter and height equations. Differences were observed in the magnitude of the coefficients estimated by the two procedures. In the volume and surface area equations there were inconsistencies in the signs as well as differences in magnitude of

the estimated coefficients. The inconsistencies do not indicate an error in the two estimation procedures but rather indicate differences in the underlying assumptions of the two procedures. Estimation of the regression coefficients depends upon the assumptions made about the variance-covariance matrix of the residuals. If the variance-covariance matrix is misspecified, the resulting estimates are unbiased but not efficient (Amemiya 1966). It should be noted that the inconsistencies in signs between the OLS and 3SLS procedures occurred in the two equations which contain current endogenous variables as explanatory variables.

Goodness-of-fit Statistics

A preferred method of model validation is to collect a new set of data which can be used to measure the prediction accuracy of the model (Reynolds et al. 1981, Snee 1977). The validity of the mathematical and theoretical assumptions used in developing and estimating the coefficients of the model is less open to question when the model gives precise estimates of the new data. In many instances the collection of new data is not practical or possible. An alternative procedure is to split the data into two independent data sets. The first data set is used to estimate the model coefficients, while the second data set is used to verify the validity of the model (Theil 1971, Snee 1977).

The OLS and 3SLS procedures were evaluated by calculating various goodness-of-fit statistics based on the estimated coefficients from each procedure. Each procedure was applied to both data sets which are as-sumed to be independent. The goodness-of-fit statistics included a fit index (R^2), mean difference ($\bar{D}$), root mean square error (RMSE), percent mean difference ($\bar{D}\%$), and percent root mean square error (RMSE%).

Table 22. Correlation matrix for the DBH equation.

PARAMETERS	INT	DBH	THT	PAR	PAR**2	ALAG/AGE	AGE	1/A**2	COMP.	DBH MICROSITE
INT	1.000	0.418	-0.420	0.070	-0.081	-0.997	0.819	-0.968	-0.342	0.098
DBH		1.000	-0.595	0.334	-0.250	-0.397	0.151	-0.556	-0.232	0.079
THT			1.000	-0.272	0.160	0.372	0.072	0.350	-0.154	0.144
PAR				1.000	-0.936	-0.080	-0.024	-0.124	-0.003	-0.127
PAR**2					1.000	0.091	-0.025	0.132	0.123	0.105
ALAG/AGE						1.000	-0.857	0.967	0.337	-0.110
AGE							1.000	-0.811	-0.326	0.201
1/A**2								1.000	0.436	-0.146
COMP.									1.000	-0.090
DBH MICROSITE										1.000

Table 23. Correlation matrix for the total height equation.

PARAMETERS	INT	DBH	THT	PAR	PAR**2	ALAG/ AGE	AGE	AGE**2	(1-ALAG/ AGE)*PAR	COMP./ SPACING
INT	1.000	-0.687	-0.027	0.112	0.019	-0.897	0.595	-0.547	-0.228	0.476
DBH		1.000	-0.580	0.169	-0.109	0.732	-0.585	0.509	-0.101	-0.275
THT			1.000	-0.255	0.192	-0.251	0.455	-0.370	0.003	-0.131
PAR				1.000	-0.850	-0.146	0.055	-0.072	-0.393	0.475
PAR**2					1.000	0.052	-0.036	0.047	-0.028	-0.339
ALAG/AGE						1.000	-0.869	0.837	0.141	-0.516
AGE							1.000	-0.988	0.006	0.360
AGE**2								1.000	-0.004	-0.346
(1-ALAG/AGE)*PAR									1.000	-0.028
COMP./SPACING										1.000

Table 24. Correlation matrix for the cubic volume equation.

PARAMETERS	INT	DBH LAGGED	THT LAGGED	PAR	PAR**2	ALAG/ AGE	AGE	1/AGE**2	(1-ALAG/ AGE)*PAR	COMP./ SPACING
INT	1.000	0.329	-0.389	-0.002	0.033	-0.996	0.739	-0.110	-0.973	-0.285
DBH LAGGED		1.000	-0.848	0.417	-0.293	-0.286	-0.181	-0.191	-0.269	-0.207
THT LAGGED			1.000	-0.449	0.329	0.329	0.242	0.126	0.241	-0.047
PAR				1.000	-0.861	-0.005	-0.224	-0.421	0.089	0.410
PAR**2					1.000	-0.021	0.159	0.020	-0.063	-0.308
ALAG/AGE						1.000	-0.787	0.104	0.976	0.259
AGE							1.000	0.045	-0.795	-0.216
1/AGE**2								1.000	0.002	-0.013
(1-ALAG/AGE)*PAR									1.000	0.367
COMP./SPACING										1.000

Table 25. Correlation matrix for the surface area equation

PARAMETERS	INT	DBH LAGGED	THT LAGGED	PAR	PAR**2	ALAG/ AGE	AGE	AGE**2	(1-ALAG/ AGE)*PAR	COMP./ SPACING
INT	1.000	0.205	-0.536	0.369	-0.128	-0.764	0.246	-0.292	0.447	0.391
DBH LAGGED		1.000	-0.848	0.427	-0.301	0.144	-0.372	0.260	-0.191	-0.187
THT LAGGED			1.000	-0.455	0.355	0.055	0.372	-0.249	0.125	-0.065
PAR				1.000	-0.860	-0.299	0.015	-0.061	-0.422	0.406
PAR**2					1.000	0.142	-0.007	0.040	0.020	-0.304
ALAG/AGE						1.000	-0.800	0.803	0.276	-0.510
AGE							1.000	-0.984	0.014	0.311
AGE**2								1.000	-0.000	-0.293
(1-ALAG/AGE)*PAR									1.000	-0.014
COMP./SPACING										1.000

Table 26. Variance inflation factors for the 3SLS growth model.

VARIABLE	D	H	V	S
INTERCEPT	8443.0	484.5	1049.1	346.0
DBH	–	–	18.8	23.5
THT	–	–	58.5	78.6
DBH LAGGED	67.1	30.4	–	–
THT LAGGED	87.1	38.3	–	–
PAR	36.2	31.5	24.1	39.4
PAR**2	18.4	13.5	12.9	14.3
ALAG/AGE	5402.6	1058.8	1196.7	764.4
AGE	51.4	764.5	37.9	985.9
AGE**2	–	210.7	9.4	274.4
1/AGE**2	789.7	–	–	–
R1P	–	12.4	203.0	13.2
COMP.	2.4	–	–	–
COMP./SP	–	3.2	3.1	3.6
DBH MICROSITE	1.1	–	–	–

Table 27 provides information on the performance of the four equations in predicting growth. The 3SLS procedure consistently performed better than the OLS procedure in estimating volumes and surface areas. The average difference for the 3SLS procedure was 0.07 cubic feet for volume and 0.13 square feet for surface area while the average error for the OLS procedure was 0.13 cubic feet and 0.17 square feet. On the average the mean difference for diameter and total height was less for the 3SLS procedure than for the OLS.

The root mean square error, RMSE, gives more weight to large deviations from the observed value more than does average differences. There was no noticable difference in the RMSE for the OLS and 3SLS procedure in the diameter and height equations. The RMSE was larger for OLS compared to the 3SLS procedure in estimating volume while the opposite was observed for the surface area equation.

Percentage of mean differences, $\overline{D}\%$, and percentages of root mean square error, RMSE%, depict the error as a proportion of the observed values. In all four equations the percentage of mean differences and percentages of root mean square error were larger in absolute value for the 3SLS procedure than for OLS.

The goodness-of-fit statistics indicate that all equations explained the variation in the data quite well using either the OLS or the 3SLS procedure.

Table 27. Evaluation of prediction equations for DBH, total height, cubic volume, and surface area estimated by OLS and 3SLS.

DATA SET[1]	ESTIMATION PROCEDURE	FIT STATISTICS [2]				
		R^2	$\overline{D}$	RMSE	$\overline{D\%}$	RMSE%
DBH EQUATIONS (IN.)						
1	OLS	0.948	−0.03	0.50	0.91	24.10
1	3SLS	0.946	−0.01	0.50	1.39	24.70
2	OLS	0.945	−0.01	0.51	1.24	17.46
2	3SLS	0.943	0.01	0.52	1.74	18.00
TOTAL HEIGHT EQUATION (FT.)						
1	OLS	0.954	−0.22	3.52	0.29	13.16
1	3SLS	0.954	−0.04	3.52	0.87	13.47
2	OLS	0.945	−0.06	3.84	0.96	14.02
2	3SLS	0.945	0.13	3.85	1.52	14.33
TOTAL VOLUME EQUATION (CU.FT.)						
1	OLS	0.978	0.13	0.74	0.41	18.28
1	3SLS	0.981	0.08	0.68	0.69	19.50
2	OLS	0.980	0.12	0.69	0.31	12.34
2	3SLS	0.982	0.06	0.65	0.60	14.32
TOTAL SURFACE AREA EQUATION (SQ.FT.)						
1	OLS	0.996	0.18	1.73	−0.02	5.80
1	3SLS	0.995	0.15	1.99	0.06	6.35
2	OLS	0.997	0.15	1.65	−0.00	4.91
2	3SLS	0.995	0.11	1.99	0.06	5.72

[1] Data set 1 (N = 2600) was used to estimate the 3SLS coefficients and data set 2 (N = 2599) was used to estimate the OLS coefficients.

[2] $I^2 = 1 - \Sigma(\hat{Y}_i - Y_i)^2 / \Sigma(Y_i - \overline{Y}_i)^2$; $\overline{D} = \frac{1}{n} \Sigma(\hat{Y}_i - Y_i)$; RMSE $= \sqrt{\frac{1}{n}\Sigma(\hat{Y}_i - Y_i)^2}$;

$\overline{D\%} = \frac{1}{n} \Sigma(\hat{Y}_i - Y_i)/Y_i$; RMSE% $= \sqrt{\frac{1}{n} \Sigma((\hat{Y}_i - Y_i)/Y_i)^2}$.

Cumulative Distribution of Errors

Knowledge of the accuracy and the efficiency of a model is often not enough. There is also a need to know the variability or precision of the estimates. Precision is measured by the standard deviation of the errors. One way to demonstrate the variability of a model is to graph the cumulative distribution of errors (Brand and Holdaway 1983). The cumulative distribution of errors for the diameter (DBH), total height (THT), cubic volume (VOL), and surface area (SUR) are depicted in Figures 19 through 22, respectively. The cumulative distribution of errors may be used to assess the likelihood of underestimating or over-estimating tree variables such as diameters or total heights by a certain amount. For example, Figure 19 shows that 97.8 percent (99 minus 1.2 percent) of the errors associated with estimating diameters were within one inch and 87 percent of the errors were within a half inch. All cumulative error distributions were skewed to the right which suggests that there is a higher likelihood of underestimating than over-estimating diameters. For example, the likelihood of underestimating diameters by more than a half inch is ten percent while there is only a three percent likelihood of overestimating diameters by more than a half inch. Similar interpretations may be made for the total height, cubic volume and surface area equations.
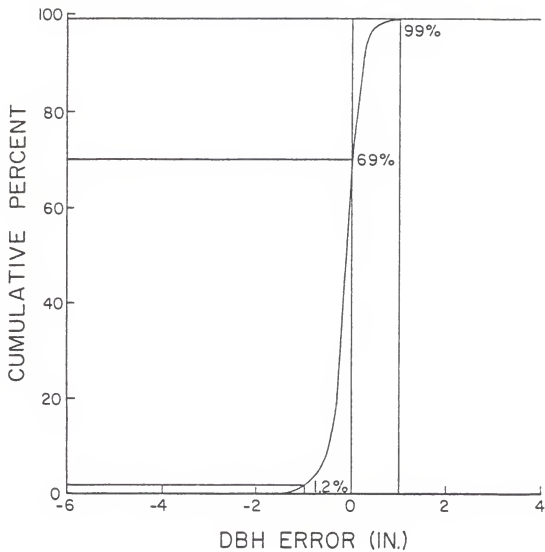
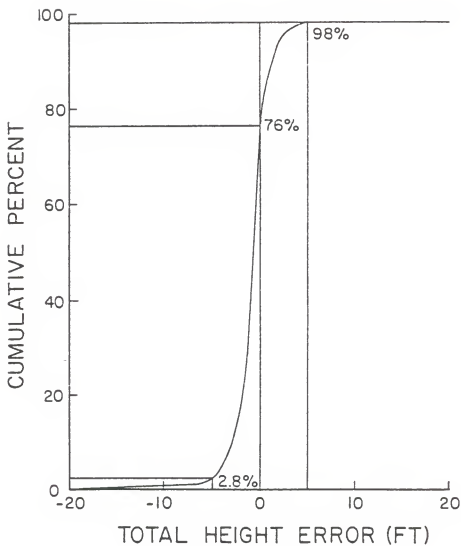Figure 19. Cumulative distribution of errors associated with the diameter breast height equation.

Figure 20. Cumulative distibution of errors associated with the total height equation.
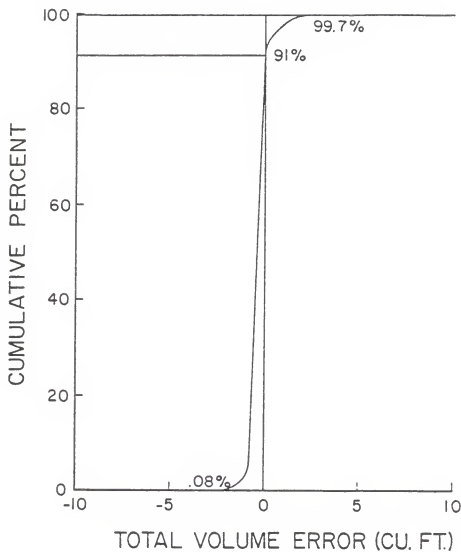
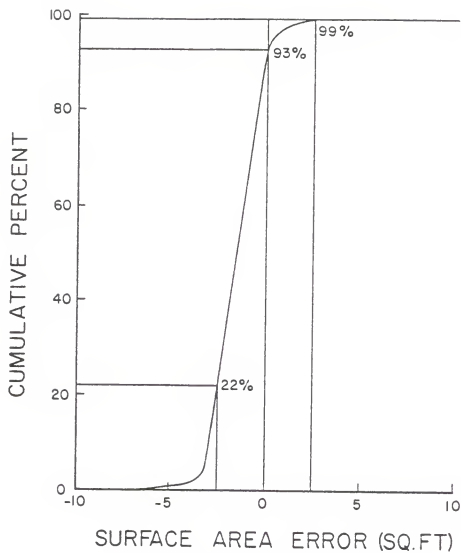Figure 21. Cumulative distribution of errors associated with the cubic volume equation.

Figure 22. Cumulative distribution of errors associated with the surface area equation.

Correlation Among Equations

Estimation of the correlations of errors across equations for the 3SLS procedure is given in Table 28. The highest correlation occurred between the volume and surface area equations (r = 0.99). This high correlation is due to the geometric relationship between surface area and cubic volume. The errors associated with the diameter equation were positively correlated with total height, cubic volume and surface area, while the errors for the total height equation were negatively correlated with cubic volume and surface area. The negative correlation associated with the height equation does not mean that total height is negatively correlated with cubic volume and surface area. Rather the height equation overestimates total height when the volume and surface area equations underestimate volume or surface area and vice-versa.

Efficiency of the OLS and 3SLS Procedures

In comparing goodness-of-fit statistics for the OLS and 3SLS procedure it is rather difficult to infer that the 3SLS is more precise than the OLS procedure in predicting diameters, total heights, cubic volumes, and surface areas. Even though both procedures (OLS and 3SLS) provide unbiased estimates of the coefficients this does not mean that the variances associated with the coefficients are efficient with respect to the Gauss-Markov Theorem.

Table 28. Correlation across equations.

Equation	Equation			
	DBH	THT	VOL	SUR
DBH	1.00	0.65	0.34	0.31
THT		1.00	−0.26	−0.29
VOL			1.00	0.99
SUR				1.00

In addition to having a model that is accurate it is also desirable that the model be efficient with respect to the estimation of the variance-covariance matrix. Since the variances associated with the estimated coefficients in the OLS and 3SLS models are independent of each other, the ratio of variances for the OLS and 3SLS procedures can be taken as an F-statistic with $\nu_1 = 2589$ and $\nu_2 = 2590$ degrees of freedom, respectively. The null hypothesis $H_0 : \sigma^2_{OLS(\hat{\beta})} = \sigma^2_{3SLS(\hat{\beta})}$, is rejected when $F_{\alpha/2\nu_1\nu_2} < F_{ij} < F_{1-\alpha/2\nu_1\nu_2}$, where $F_{\alpha/2\nu_1\nu_2}$ and $F_{1-\alpha/2\nu_1\nu_2}$ denote the upper $\alpha/2$ and lower $1-\alpha/2$ percentage points of the F-distribution, with ν_1 and ν_2 degrees of freedom, and F_{ij} is the ratio of variances associated with the jth coefficient in the ith equation. The relationship between the upper and lower tail percentage points is given by:

$$F_{1-\alpha/2\nu_1\nu_2} = \frac{1}{F_{\alpha/2\nu_1\nu_2}} .$$

Since there are no F-tables available for large sample sizes (n > 500) a conservative estimate of the rejection region (ν=500) was used for hypothesis testing.

In the diameter (D) and height (H) equation all but four of the variance-ratios were significantly greater than one at the 0.10 level (Table 29). If F-tables were available for large sample sizes (n > 500] at least two of the nonsignificant variance-ratios would be significantly greater than one since $F_{\alpha/2\nu_1\nu_2}$ approaches one for all levels of α as n goes to infinity. The results indicate that 3SLS is more efficient in estimating the coefficients of the diameter and height equations than OLS.

In the volume (V) and surface area (S) equations all variance – ratios were significantly different from one at the 0.02 level; six and seven of the variance ratios, respectively, were significantly less than one. At first glance it would seem that OLS in more efficient than 3SLS in estimating certain parameters, but in fact the results are due to a specification error in the variance-covariance matrix. The OLS procedure does not take into consideration the correlation of errors among equations nor the presence of endogenous variables as explanatory ones.

Table 29. Efficiency of the OLS and 3SLS estimation procedure for DBH
(D), total height (H), cubic volume (V), and surface area (S)
equations.

| PARAMETER | $\hat{\sigma}^2_{OLS}$ / $\hat{\sigma}^2_{3SLS}$ | | | |
	D	H	V	S
INTERCEPT	1.36**	1.43**	3.47**	0.61**
DBH	---	---	0.35**	0.35**
TOTAL HEIGHT	---	---	0.28**	0.28**
DBH LAGGED	1.02	1.24**	---	---
THT LAGGED	1.08	1.29**	---	---
PAR	1.02	1.34**	0.62**	0.60**
PAR**2	1.00	1.21*	0.64**	0.63**
ALAG/AGE	1.36**	1.60**	3.61**	1.59**
AGE	1.31**	1.69**	1.45**	3.85**
AGE**2	---	1.70**	---	4.84**
1/AGE**2	1.42**	---	4.83**	---
(1-ALAG/AGE)*PAR	---	2.02**	0.77**	0.74**
COMP.	1.69**	---	---	---
COMP./SP	---	1.74**	0.87**	0.80**
DBH MICROSITE	3.22**	---	---	---

* = significantly different (α=0.10) according to a two-tailed F-test;
   $\nu_1 = \nu_2 = 500$.
** = significantly different (α=0.02) according to a two-tailed F-test;
   $\nu_1 = \nu_2 = 500$.

SUMMARY AND CONCLUSIONS

A growth model is presented to depict the effects of competition and available growing space on the growth and development of individual slash pine trees over time. The model predicts the cumulative growth of individual tree diameters, total height, total cubic volume, and total bole surface area using a modified form of the Verhulst growth function.

Two estimation procedures, (1) Ordinary Least Squares (OLS) and (2) Three Stage Least Squares (3SLS) are considered under two assumptions concerning the residual errors (contemporaneous covariance and serial correlation). The OLS procedure assumes that the variables in the model simultaneously satisfy certain assumptions regarding the equality of variances among regressors as well as the independence of the explanatory variables from the residual errors. Likewise, the procedure does not take into consideration the interrelationship among equations. The 3SLS procedure, on the other hand, assumes a correlation among equations (contemporaneous covariance) and simultaneously estimates the coefficients associated with the system of equations.

To assess the effects of size and distribution of vacancies on individual tree growth, Hopkins' modeified coefficient of aggregation was used to analyze the spatial distribution of healthy and dead trees four years after planting. This age was selected because most of the mortality had occurred by this time. Healthy trees were regularly

distributed throughout the study area while dead trees resembled a random spatial pattern. The spatial distribution of healthy and dead trees varied significantly according to spacing and plot location. The results of the spatial analysis indicate no adverse affects on the growth and development of the remaing trees in the study area due to mortality.

Often, in pooling data across a wide range of planting densities, it is difficult, if not impossible to capture all of the important cross-sectional effects. Furthermore, data obtained by repeated measurements will contain correlated errors. Unequal variances may also be present in the data. To test the assumptions regarding the cross-sectional and time-series error component of the growth model, Maximum Likelihood (ML) estimates of the parameter vector and variance-covariance matrix were obtained by searching over all values of ρ , $(0 < \rho < 1)$, and choosing the corresponding parameter set which maximizes the likelihood function. Here, ρ is the ratio of the cross-sectional and time-series error component to the total model variance. All ML estimates of ρ for the four equations were zero, which implies that the model variance arises from random sampling errors and not from a cross-sectional or time-series error component. As a result, the standard 3SLS procedure was used to estimate the parameters associated with the growth model.

The growth model was evaluated using data from a spacing study established in 1962. Slash pine seedlings were hand planted in twelve one-acre blocks measuring 2.5 x 4 chains at six different spacings; 5x10, 7x10, 9x10, 11x10, 13x10, and 15x10 feet. Tree measurements were collected on a subsample of trees, from each plot, approximately every

two years starting six years after planting. Measurements included diameter at breast height, merchantable height to a four inch top, total height, position in the canopy, and number of primary and secondary competing trees. Volumes and surface areas were estimated from regression equations developed from dendrometry data obtained from the study area.

The data were split into two sets; the first set (N=2599) was used to identify the structural form of the growth model and obtain OLS estimates of the coefficients, while the second data set (N=2600) was used to estimate the coefficients associated with the structural system of equations using 3SLS. The validity of the two models were assessed by comparing data-fit and prediction accuracy. The efficiency of the two procedures were evaluated by taking the ratio of variances for the OLS and 3SLS procedures as an F-statistic. The signs of the coefficients were also compared with theoretical expectations.

The OLS and 3SLS estimates were consistant with respect to the signs of the coefficients for the diameter and height equation. However, differences were observed in the magnitude of the coefficients. Two coefficients in the diameter equation had wrong signs. The signs indicate that increasing competition increases diameter growth, while an increase in site decreases diameter growth. In the volume and surface area equations inconsistencies were observed in the signs of the coefficients as well as differences in the magnitude of the estimated coefficients. These inconsistencies do not indicate an error in the estimation procedures, but rather indicate differences in the underlying assumptions regarding the structure of the variance-covariance matrix used in the estimation procedure.

The 3SLS procedure consistently performed better than OLS in estimating volumes and surface areas. For example, the average difference for 3SLS was 0.07 cubic feet for volume and 0.13 square feet for surface area, while the average error for OLS was 0.13 cubic feet and 0.17 square feet. On the average, the mean difference for diameters and total height was less for 3SLS than for OLS.

The cumulative distribution of errors were skewed to the right for the four equations; this suggests that there is a higher likelihood of underestimating than overestimating. For example, the likelihood of underestimating total height by three feet is ten percent while there is only a three percent likelihood of overestimating total height by more than three feet.

In comparing goodness-of-fit statistics for the OLS and 3SLS procedures it is rather difficult to infer that 3SLS is more precise than OLS in estimating diameter, total height, total cubic volume, and total surface area. Even though both procedures yield unbiased estimates of the coefficients, this does not ensure that the estimates are efficient. Results of this study indicate that 3SLS was more efficient than OLS in estimating diameter and total height. In the volume and surface area equations OLS was more efficient than 3SLS in estimating certain parameters. This difference is due to a specification error in the variance-covariance matrix and the presence of endogenous variables as explanatory ones.

In conclusion, it is suggested that OLS though quite adequate for estimating model parameters, should not be used in constructing confidence intervals or hypothesis testing with data containing contemporaneous covariances and/or autocorrelated errors. To ensure

asymptotically efficient parameter estimates, 3SLS is a better procedure to simultaneously estimate the coefficients associated with a system of equations.

FUTURE RESEARCH

An important consideration in stand management decisions is the variability and uncertainty in the growth and development of a given stand over time (Lembersky and Johnson, 1975). Stand management in terms of thinning or harvest option at one point in time influence the alternatives feasible in the future. Random or chance events (e.g. levels of disease infection and mortality) will influence present and future decisions.

In the past the development of management strategies for infected slash pine plantations has been based primarily on the level of infestation (hazard rating), level of stocking, and the age of the stand. Two aspects of this problem that have not been addressed pertain to the spatial distribution of the disease and the economic implication of managing infected stands.

If the spatial distribution of the disease related mortality is random or uniformly distributed throughout a given stand, the additional growing space would be similar to a random or systematic thinning which may increase the productivity of the stand over time. If the spatial distribution of the mortality is aggregated the residual trees may not be able to utilize the additional growing space and may result in a decrease in the productivity of the stand (Reich 1980).

The development of management strategies for infected stands should be evaluated and based on an individual stand basis. Management decisions should not only take into consideration the level of infestation but the economics of managing the stand (e.g. interest rate, management costs, and timber prices), management objectives (pulpwood and/or sawtimber), planting density, site index, and management alternatives (e.g. row thinning, as is, thinning from below, and sanitation thinning). All of these factors play a significant role in determining the optimal rotation age and present net value of a given stand.

In recent years, interest has developed in defining and solving sequential decision problems associated with forest management. The process is a multistage or sequential decision problem characterized by a sequence of decisions made chronologically. Following each decision a chance event occurs which will, in turn, influence the next decision (Bubnick et al. 1977). A widely used approach for solving sequential decision problems is dynamic programming for solving joint optimization of the thinning plan and rotation length. Recently the impacts of regeneration cost, initial stocking level, site, quality premiums and variable logging costs on optimal thinning and harvest scheduling have been solved via this approach (Brodie et al. 1978).

Most applications of dynamic programming for forest stand analysis have incorporated stand-level growth models which implicitly assume some type of diameter distribution structure. This is an important assumption since a stand's diameter distribution defines both its current value and expected growth.

If an optimal thinning and final harvest scheduling model is to truly assist in stand management, the model should allow for disease infection and growth losses to affect the marginal value growth of the stand. A Dynamic Simulation Model (DSM) incorporating an individual tree growth model can be utilized to develop alternative management strategies correlated to a diseased infection level and its spatial distribution. Such a model can develop growth estimates which incorporate individual tree competition and the effect of individual tree removals (either through cut or mortality) especially as the spatial distribution is varied.

Until the effects of various management practices on growth, yield and susceptibility of a stand are quantified, realistic assumptions will have to be made to adjust certain stand growth coefficients accordingly. Depending upon the degree of sophistication of the growth and yield models, they may include such variables as the growth rate of individual trees, site competitive status of trees, one or more random tree components representing the microsite and/or genetic variability, or susceptibility to certain forest pests. The information obtained would not only assist land managers in developing rational and contingency plans, but would also give them an estimate of the expected volume loss for a given set of conditions.

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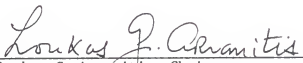Watson, G.S. 1955. Serial correlation in regression analysis. Biometrica 42:327-341.

BIOGRAPHICAL SKETCH

Robin Michael Reich was born October 24, 1954, in Bayreuth, Germany, while his father was stationed in Germany with the Second Cavalry Regiment, U. S. Army. His younger years reflected moves to Nurnberg, Stuttgart and Berlin. In 1960 his family returned to the U. S. to Fort Sam Houston, San Antonio, Texas, followed by another transfer to Fort Benning, Georgia, in 1964. His family settled in Winter Park, Florida, in 1966 where Robin graduated from high school and received his AA degree from Seminole Community College.

Robin entered the University of Florida in September, 1975, where he received his Bachelor of Science in June, 1978, and his Master of Science in August, 1980, in forest resources and conservation. Robin received an award of excellence for graduate research in October, 1980, and a certificate of recognition from NASA for creative development of a technical innovation in remote sensing, June, 1981.
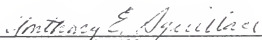
Robin is a member of Phi Kappa Phi and Xi Sigma Pi, honorary societies at the University of Florida.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.
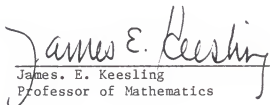
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This dissertation was submitted to the Graduate Faculty of the School of Forest Resources and Conservation in the College of Agriculture and to the Graduate Council, and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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